Multistage Model for Distribution Expansion Planning With Distributed Generation—Part I: Problem Formulation

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***Abstract*—This paper presents a model for use in the problem of multistage planning of energy distribution systems including distributed generation. The expansion model allows alternatives to be considered for increasing the capacity of existing substations, for installing new ones, for using distributed generation, and for the possible change to feeders in terms of addition and removing feeders sections; combining, subdividing, and load transfer between feeders; and replacement of conductors. The objective function to be minimized is the present value of total installation costs (feeders and substations), of operating and maintaining the network, and of distributed generation. The model takes operational constraints on equipment capacities and voltage limits together into account with logical constraints, aimed at reducing the search space. This paper presents: 1) an extension to the linear disjunctive formulation to represent the inclusion, exclusion, and replacement of branches and 2) a generalization of constraints related to the creation of new paths which can be applied in more complex topologies. The resulting mixed integer linear model allows the optimal solution to be found using mathematical programming methods, such as the branch-and-bound algorithm. The validity and efficiency of the model are demonstrated in Part II of this paper.**

***Index Terms*—Distributed generation, power distribution, power distribution economics, power distribution planning.**

NOMENCLATURE

A. Sets

Ψ^F, Ψ_i^F	Branches of the fixed network and defined changes for each branch i.
Ψ^R, Ψ_j^R	Branches of the replacement network and alternatives for each branch j.
Ψ^A, Ψ_k^A	Branches of the addition network and alternatives for each branch k.
Ψ^S, Ψ_l^S	Nodes of the existing and candidate substation and alternatives for expanding the capacity of the node l.
Ψ^D	Load nodes.
Ψ^G	Distributed generation nodes.

B. Binary variables

$x_{j,t}^{RJ}$	Alternative RJ of the replacement branch j at stage t.
$x_{k,t}^{AK}$	Alternative AK of the addition branch k at stage t.
$x_{l,t}^{S0}$	Fixed costs for installation or expansion of substation node l at stage t.
$x_{l,t}^{SL}$	Alternative SL for expansion of substation node l at stage t.
$\mathbf{x}_t$	Investment vector at stage t.
$y_{i,t}^{FI}$, $y_{\max,i,t}^{FI}$	Alternative FI for utilization of the branch i at stage t: $y_{\max,i,t}^{FI}=1$ means that the alternative FI is available.
$y_{j,t}^{R0}$, $y_{j,t}^{RJ}$	Alternatives $R0$ and RJ for utilization of the replacement branch j at stage t.
$y_{k,t}^{AK}$	Alternative AK for utilization of the addition branch k at stage t.

C. Continuous variables

$f_{i,t}^F$, $f_{\max,i}^{FI}$	Current of branch i in the fixed network at stage t.
$f_{j,t}^R$, $f_{\max,j}^{R0}$, $f_{\max,j}^{RJ}$	Current of branch j in the replacement network at stage t and their maximum capacities $R0$ and RJ.
$f_{k,t}^A$, $f_{\max,k}^{AK}$	Current of branch k in the addition network at stage t and its maximum capacity AK.
$\mathbf{f}_t^F$, $\mathbf{f}_t^R$, $\mathbf{f}_t^A$	Current vectors for the fixed, replacement, and addition branches at stage t.
$g_{l,t}^S$, $g_{\max,l,t}^S$, $g_{\max,l}^{SL}$	Generation at node l at stage t and the substations' maximum capacity.
$r_{m,t}$	Nodal-load shedding at stage t.
$\mathbf{r}_t$	Vector of nodal-load shedding at stage t.
$g_{n,t}^G$, $g_{\max,n,t}^G$	Distributed generation at node n at stage t and its maximum capacity.

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$\mathbf{g}_t$	Column vector of the nodal injection at stage t.
$\mathbf{V}_t$, $\mathbf{V}_{\min}$, $\mathbf{V}_{\max}$	Column vectors of the nodal voltages at stage t and their minimum and maximum limits.
$c_t^{\text{inv}}(\mathbf{x}_t)$	Investment cost at stage t.
$c_t^{\text{oper}}(\mathbf{r}_t, \mathbf{g}_t)$	Operation cost at stage t.

D. Parameters

$d_{m,t}$	Nodal load at node m at stage t.
$\mathbf{d}_t$	Vector of nodal load at stage t.
$\mathbf{S}^F$	Node-branch incidence matrix for the fixed network.
$\mathbf{S}^R$	Node-branch incidence matrix for the replacement network.
$\mathbf{S}^A$	Node-branch incidence matrix for the addition network.
Z_i^{FI}	Impedance of the alternative FI related to the branch i of the fixed network.
Z_j^{R0}, Z_j^{RJ}	Initial and alternative impedances related to the branch j of the replacement network.
Z_k^{AK}	Impedance of the alternative AK of the addition network.
M	Big number used in the inequalities of the disjunctive constraints.
N_t	Number of network nodes at stage t, excluding substations nodes.
δ_t^{inv}, δ_t^{oper}	Present value factors for the investment and operation costs at stage t.
O_i^{FI}	Operation cost of the alternative FI related to the branch i.
C_j^{RJ}, O_j^{RJ}	Investment and operation costs of the alternative RJ related to the replacement branch j.
C_k^{AK}, O_k^{AK}	Investment and operation costs of the alternative AK related to the addition branch k.
C_l^{S0}, C_l^{SL}	Fixed and variable costs of a substation located at node l.
C_m^D	Load shedding cost at node m.
C_n^G	Distributed generation cost at node n.
B, B_t	Total budget and budget at stage t.
T	Number of stages of the planning horizon.
I	Rate of interest for a time period.
$p(t)$	Number of time periods up to stage t counted from a given time reference (month or year).
$\Delta p(t)$	Time periods of stage t.

I. Introduction

THE problem of expansion planning to a distribution system consists of determining the capacity, siting, and timing of installation of new distribution equipment, taking capacity restrictions on feeders, voltage drop, and demand forecasts into account [1]–[4]. Initially, a number of authors solved a simplified form of this problem, using a static planning model with a fixed time horizon [5]–[8]. Their work resulted in a formalization of the problem in a single stage, in which the resources are introduced at one single time step over the planning horizon. In general, a short-term planning horizon has been used so that those investments are selected which correspond to the network's immediate needs, since the uncertainty in forecasts tends to increase as the time horizon increases.

Subsequently, the problem was adapted to deal with a long-term time horizon [9]–[13]. This approach resulted in a multistage formulation of the problem in which resources needed for the planning horizon can be distributed according to the requirements determined at each stage. Network operators can thereby accommodate the gradually increasing demand at minimum cost, using a long-term planning horizon. The investments needed for the initial steps are effectively executed while the investments for later stages are reevaluated in the future with the use of updated forecasts. The planning horizon is therefore dynamically advanced, with the initial stage always coinciding with the time of execution (month or year).

The methods used to solve the expansion planning problem can be divided into two categories: methods of mathematical programming and heuristic methods, including specialist systems and evolutionary algorithms. Among the methods of mathematical programming, the most widely used include mixed integer programming [6], [7], [14]–[16], nonlinear programming [17]–[19], dynamic programming [20], [21], and linear programming [22]. In this approach, it is possible to represent the main restrictions explicitly (Kirchhoff's laws, equipment capacities, voltage drop, and budget) and to minimize fixed and variable costs arising from installation and substitution of equipment. Where mixed integer programming is used, practical considerations frequently limit the number of solutions and make the associated combinatorial problems computationally tractable [23]. This, together with the possibilities both of guaranteeing optimality and of using the computers resources currently available, makes the approach very attractive.

Since 1980, much effort has been directed toward solving the problem of planning distribution by the use of heuristic algorithms, which came to provide an alternative to mathematical programming. Heuristic methods gained attention because they can work in a straightforward fashion with nonlinear constraints and objective function, although there is no guarantee that an optimum solution can be found. However, in this approach, it is also easy to introduce aspects, such as losses, reliability, and uncertainties. Notable among heuristic methods are the branch-exchange algorithms [8], [13], [24], [25] and algorithms based on evolutionary computation [26]–[29]. Other heuristic methods that have been used for the problem include specialist systems [30], [31], the ant colony [32], simulated annealing [33], and tabu search [34], [35].

This paper obtains the optimum solution to multistage planning problem through the use of mathematical programming methods. The model considers the possible ways in which nodes and branches of the distribution system may be modified, together with the use of distributed generation and various operational and financial constraints. Modifications to nodes include: installation of new substations, upgrading substation capacities, utilization of distributed generation, and load shedding. Modifications to branches include installation, replacement, and removal of feeder segments. The paper also extends the disjunctive approach typically used in transmission expansion planning with a dc load-flow network model [36], [37]. In formulating transmission and distribution planning problems, nonlinear constraints are expressed in terms of the product of binary and continuous variables. In the classical approach, only the inclusion or noninclusion of a branch is represented by means of a disjunctive formulation; with the approach given in this paper, the removal and replacement of branches is also dealt with in this way.

To reduce the size of search space in the mixed integer problem in which the planned expansion is cast, the following were introduced: 1) logical constraints which describe investment limitations; 2) fencing constraints obtained from the Kirchhoff's currents laws (KCL); 3) constraints on new paths [38], which are generalized to allow their use with more complex topologies.

This paper is organized as follows. Section II shows how the optimization problem is modeled, with details of the objective function and constraints imposed. Section III describes the load and network models. This paper ends with a presentation of conclusions. In order to illustrate the effectiveness of the proposed model, four different planning situations of a medium-voltage distribution network are presented in the second part of this paper [39].

II. Problem Formulation

The problem of distribution system planning with a long-term horizon has been modeled taking the following factors into account:

- the distribution network is composed of nodes at which loads and sources are concentrated, and branches forming connections between nodes, representing the feeders;
- the planning horizon is divided into T stages of known duration, with the variables in the problem being associated with each stage;
- two continuous variables are associated with each node: one is the absolute value of nodal voltage and the other is the current injection; one continuous variable, current flow, is associated with each network branch;
- in each stage to the planning horizon, nodes may be modified by increasing the capacity and installing new substations; branches may be modified by conductors replacing or by adding branch connecting nodes not previously connected;
- possible alterations to the network nodes and branches constitute a set of investment alternatives to be used in solving the network expansion problem;
- associated with the execution of each investment alternative throughout the stages, there is a binary variable x_t, having the value one when the alternative is selected at stage t and the value zero, otherwise;
- each type of alteration has investment costs associated with replacing one branch by another one (C_j^{RJ}); with adding a new branch (C_k^{AK}); and with increasing the capacity and installing a new substation at a node (C_l^{S0} and C_l^{SL});
- associated with the use of the available network branches, there are binary variables y_t, having the value one when the alternatives are used at stage t and the value zero, otherwise;
- all network branches have associated operational and maintenance costs (O_i^{FI}, O_j^{R0}, O_j^{RJ}, and O_k^{AK});
- the available capacities for distributed generation are taken into account when defining the planned network expansion;
- in each stage, current injections, nodal voltages, and current flows satisfy Kirchhoff's laws;
- load is represented by current injections with known values for each stage;
- limits on conductor capacities, substation capacities, and availability of distributed generation are taken into account at each stage;
- voltage drops in the distribution network are calculated as the product of branch current and branch impedance;
- objective function to be minimized is the present value of investment and operational costs;
- limits are set on total investment (B) and on investment at stage $t(Bt)$.

Variables associated with the alterations defined for branch i of the fixed network are written with the superscript FI. Variables associated with possible alterations to branch j of the replacement network are written with the superscript RJ. Variables associated with possible alterations to branch k of the addition network are written with the superscript AK. Variables associated with expansion of the substation at node l are written with the superscript SL. With this representation, the number of alterations associated with each branch or node of the network can be defined, independent of the alterations defined for other branches or nodes. Thus, some network branches may have just one possible alteration, others may have two, others three, and so on.

Variables showing whether investments should be selected are grouped according to the type of alteration proposed: replacing the conductor of an existing branch (variables $x_{j,t}^{RJ}$); addition of a new branch (variables $x_{k,t}^{AK}$); installation of a new substation or increasing the capacity of a substation at a node (variables $x_{l,t}^{S}$ and $x_{l,t}^{SL}$, respectively). For each stage, the use of available resources (already existing or previously installed) is associated with a set of binary variables ($y_{i,t}^{FI}$, $y_{j,t}^{RJ}$, and $y_{k,t}^{AK}$) with values one when the respective alternative was used at stage t.

Alterations to network branches defined *a priori* (addition, replacement, or removal) are easily incorporated in the model by means of limits on variables $y_{i,t}^{FI}$ defined for each stage. For example, if alternative $F2$ of branch 5 was available during stage three, the limits would be given by $y_{\max,5,1}^{F2} = y_{\max,5,2}^{F2} = 0$ and $y_{\max,5,3}^{F2} = 1$. Thus, it would not be possible to use the alternative $F2$ in the first two stages. Similarly, *a priori* alterations defined for network nodes (substation capacities) can be introduced into the model by means of limits on existing capacities $(g_{\max,l,t}^{S})$ defined for each stage.

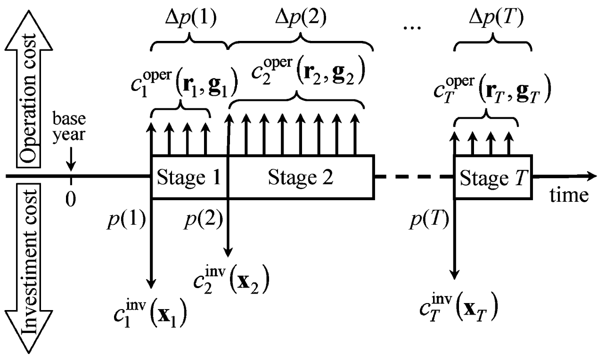

Fig. 1. Instants at which operational and investment costs come into effect in the multistage problem.

Thus, the expansion planning of distribution networks becomes a mixed integer problem (MIP), defined as follows.

A. Objective Function

The objective function for the network expansion problem has two parts: the investment cost $(c_t^{\text{inv}}(\mathbf{x}_t))$ and the operational cost $(c_t^{\text{oper}}(\mathbf{r}_t, \mathbf{g}_t))$, as shown in Fig. 1.

The investment cost is determined at the beginning of each stage and is given by the cost of altering network branches (by changing cross-sections of already-existing feeder sections or by installing new feeder sections denoted by C_j^{RJ} and C_k^{AK}) and network nodes (by increasing the capacity of existing substations or by installing new ones, denoted by C_l^{S0} and C_l^{SL}). The cost of operation is considered at the beginning of each stage period and corresponds to the annual operational and maintenance cost of network branches that are in use (O_i^{FI}, O_j^{R0}, O_j^{RJ}, and O_k^{AK}), to energy that is not supplied (C_m^D) and to the additional cost of energy supplied from distributed generation (C_n^G).

When the planning horizon is divided into T stages, the objective function to be minimized is the present value of costs distributed through time, and is given by the following expression:

$$C(\mathbf{x},\mathbf{r},\mathbf{g}) = \sum_{t=1}^{T}\left[\delta_t^{\text{inv}} c_t^{\text{inv}}(\mathbf{x}_t) + \delta_t^{\text{oper}} c_t^{\text{oper}}(\mathbf{r}_t,\mathbf{g}_t)\right] \quad (1)$$

$$c_t^{\text{inv}}(\mathbf{x}_t) = \sum_{j\in\Psi^R}\sum_{J\in\Psi_j^R} C_j^{RJ} x_{j,t}^{RJ} + \sum_{k\in\Psi^A}\sum_{K\in\Psi_k^A} C_k^{AK} x_{k,t}^{AK} + \sum_{l\in\Psi^S}\left(C_l^{S0} x_{l,t}^{S0} + \sum_{L\in\Psi_l^S} C_l^{SL} x_{l,t}^{SL}\right) \quad (1.1)$$

$$c_t^{\text{oper}}(\mathbf{r}_t,\mathbf{g}_t) = \sum_{i\in\Psi^F}\sum_{I\in\Psi_i^F} O_i^{FI} y_{i,t}^{FI} + \sum_{j\in\Psi^R}\left[O_j^{R0} y_{j,t}^{R0} + \sum_{J\in\Psi_j^R} O_j^{RJ} y_{j,t}^{RJ}\right] + \sum_{k\in\Psi^A}\sum_{K\in\Psi_k^A} O_k^{AK} y_{k,t}^{AK} + \sum_{m\in\Psi^D} C_m^D r_{m,t} + \sum_{n\in\Psi^G} C_n^G g_{n,t}^G \quad (1.2)$$

$$\delta_t^{\text{inv}} = \frac{1}{(1+I)^{p(t)}} \quad (1.3)$$

$$\delta_t^{\text{oper}} = \sum_{p=p(t)}^{p(t)+\Delta p(t)-1} \frac{1}{(1+I)^p}. \quad (1.4)$$

The use of fictitious current injections $r_{m,t}$ to represent load shedding ensures that feasible solutions always exist even under investment constraints. These constraints can therefore be defined for each stage and for the planning period as a whole, without compromising the existence of feasible solutions to the problem.

B. Constraints in the Problem

The constraints are divided into four blocks arising from Kirchhoff's laws, from operational limits on equipment, and from the availability of resources (financial limits). The first block of constraints comes from imposing KCL at all stages

$$\mathbf{S}^F\mathbf{f}_t^F + \mathbf{S}^R\mathbf{f}_t^R + \mathbf{S}^A\mathbf{f}_t^A + \mathbf{g}_t + \mathbf{r}_t = \mathbf{d}_t \quad \forall t = 1,\cdots,T. \quad (2)$$

The second block of constraints comes from applying the Kirchhoff's voltages law (KVL), for all $t = 1,\cdots,T$. For the network's fixed branches, for branches that are candidates for conductor replacement, and for branches that could be added, we have

$$Z_i^{FI} f_{i,t}^F + [\mathbf{S}^F]_{\text{row}i}^{\text{T}}\mathbf{V}_t = 0 \ \left\{\forall i\in\Psi^F, I\in\Psi_i^F | y_{i,t}^{FI} = 1\right\} \quad (3)$$

$$Z_j^{R0} f_{j,t}^R + [\mathbf{S}^R]_{\text{row}j}^{\text{T}}\mathbf{V}_t = 0 \ \left\{\forall j\in\Psi^R | y_{j,t}^{R0} = 1\right\} \quad (4)$$

$$Z_j^{RJ} f_{j,t}^R + [\mathbf{S}^R]_{\text{row}j}^{\text{T}}\mathbf{V}_t = 0 \ \left\{\forall j\in\Psi^R, J\in\Psi_j^R | y_{j,t}^{RJ} = 1\right\} \quad (5)$$

$$Z_k^{AK} f_{k,t}^A + [\mathbf{S}^A]_{\text{row}k}^{\text{T}}\cdot\mathbf{V}_t = 0 \ \left\{\forall k\in\Psi^A, K\in\Psi_k^A | y_{k,t}^{AK} = 1\right\} \quad (6)$$

where the superscript T indicates the matrix transposition. It should be noted that the existence of constraints (3)–(6) depends on the variables $y_{i,t}^{FI}$, $y_{j,t}^{R0}$, $y_{j,t}^{RJ}$, and $y_{k,t}^{AK}$, which determine whether resources are used. The constraints only operate when the respective utilization variable has value one (indicating that it is used in the stage). This condition is implemented by multiplying the constraint by the corresponding utilization variable. Nonlinearities are thereby introduced into the model through multiplication of the utilization variables by the currents ($f_{i,t}^F$, $f_{j,t}^R$, and $f_{k,t}^A$) and by the voltages V_t. This nonlinearity is avoided by adopting an extension of the linear disjunctive model successfully used in planning the expansion of transmission networks [36], [37]. With this novel formulation, the resulting problem becomes linear and can be solved directly using classical optimization methods without applying decompositions or heuristic methods. In the application proposed in this paper, the following functional relationships are used.

- For each branch, various possible configurations are considered in terms of different cross sections and structure type with only the most appropriate being selected.
- At any stage, the removal of obsolete branches is considered, with feeders divided into sections.

Equations (3)–(6) are therefore substituted by the following disjunctive versions given by:

$$\begin{aligned}&\left|Z_i^{FI} f_{i,t}^F + [\mathbf{S}^F]_{\text{row}i}^{\text{T}} \mathbf{V}_t\right| \\ &\quad \le M\left(1 - y_{i,t}^{FI}\right) \ \{\forall i \in \Psi^F, I \in \Psi_i^F\} \qquad (3.1)\\ &\left|Z_j^{R0} f_{j,t}^R + [\mathbf{S}^R]_{\text{row}j}^{\text{T}} \mathbf{V}_t\right| \\ &\quad \le M\left(1 - y_{j,t}^{R0}\right) \ \{\forall j \in \Psi^R\} \qquad (4.1)\\ &\left|Z_j^{RJ} f_{j,t}^R + [\mathbf{S}^R]_{\text{row}j}^{\text{T}} \mathbf{V}_t\right| \\ &\quad \le M\left(1 - y_{j,t}^{RJ}\right) \ \{\forall j \in \Psi^R, J \in \Psi_j^R\} \qquad (5.1)\\ &\left|Z_k^{AK} f_{k,t}^A + [\mathbf{S}^A]_{\text{row}k}^{\text{T}} \mathbf{V}_t\right| \\ &\quad \le M\left(1 - y_{k,t}^{AK}\right) \ \{\forall k \in \Psi^A, K \in \Psi_k^A\}. \qquad (6.1)\end{aligned}$$

When a variable y in (3.1)–(6.1) is zero, the respective constraint is relaxed, since M is big enough for both inequalities to always be satisfied for the possible values of V_t. When y has value one, the inequalities (3.1) to (6.1) work in the same way as their counterpart inequalities in (3)–(6).

The third block of constraints includes the operational limits on equipment and the limits on available investment. Limits on branch currents for $t = 1, \cdots, T$ depend on the use of available resources and are given by

$$\left|f_{i,t}^F\right| \le \sum_{I \in \Psi_i^F} y_{i,t}^{FI} f_{\max,i}^{FI} \quad \{\forall i \in \Psi^F\} \qquad (7)$$

$$\left|f_{j,t}^R\right| \le y_{j,t}^{R0} f_{\max,j}^{R0} + \sum_{J \in \Psi_j^R} y_{j,t}^{RJ} f_{\max,j}^{RJ} \quad \{\forall j \in \Psi^R\} \qquad (8)$$

$$\left|f_{k,t}^A\right| \le \sum_{K \in \Psi_k^A} y_{k,t}^{AK} f_{\max,k}^{AK} \quad \{\forall k \in \Psi^A\}. \qquad (9)$$

For the substations, the limits to current injections for $t = 1, \cdots, T$ depend on available capacity and on investments in increasing the substation capacity undertaken at each stage, and are given by

$$0 \le g_{l,t}^S \le g_{\max,l,t}^S + \sum_{L \in \Psi_l^S} \left(\sum_{\tau=1}^{t} x_{l,\tau}^{SL}\right) g_{\max,l}^{SL} \quad \{\forall l \in \Psi^S\}. \qquad (10)$$

For nodes with capacity for distributed generation, current injections limits for $t = 1, \cdots, T$ depend on the capacity available at each stage, and are given by

$$0 \le g_{n,t}^G \le g_{\max,n,t}^G \quad \{\forall n \in \Psi^G\}. \qquad (11)$$

The remaining operational limits are the maximum load shedding and the range of acceptable values for the nodal voltages at substations and at nodes where loads or generation are installed for $t = 1, \cdots, T$

$$\begin{aligned}0 \le r_{m,t} \le d_{m,t} \quad &\{\forall m \in \Psi^D\} \qquad (12)\\ V_{\min,l} \le V_{l,t} \le V_{\max,l} \quad &\{\forall l \in \Psi^S\} \qquad (13)\\ V_{\min,m} \le V_{m,t} \le V_{\max,m} \quad &\{\forall m \in \Psi^D\} \qquad (14)\\ V_{\min,n} \le V_{n,t} \le V_{\max,n} \quad &\{\forall n \in \Psi^G\}. \qquad (15)\end{aligned}$$

The limits on investment for each stage and for the entire planning horizon, are given by the following expressions:

$$\begin{aligned}&\sum_{j \in \Psi^R} \sum_{J \in \Psi_j^R} C_j^{RJ} x_{j,t}^{RJ} + \sum_{k \in \Psi^A} \sum_{K \in \Psi_k^A} C_k^{AK} x_{k,t}^{AK} \\ &\quad + \sum_{l \in \Psi^S} \left(C_l^{S0} x_{l,t}^{S0} + \sum_{L \in \Psi_l^S} C_l^{SL} x_{l,t}^{SL}\right) \\ &\quad \le B_t \quad \forall t = 1, \cdots, T \qquad (16)\end{aligned}$$

$$\begin{aligned}&\sum_{t=1}^{T} \left[\delta_t^{\text{inv}} \left(\sum_{j \in \Psi^R} \sum_{J \in \Psi_j^R} C_j^{RJ} x_{j,t}^{RJ} + \sum_{k \in \Psi^A} \sum_{K \in \Psi_k^A} C_k^{AK} x_{k,t}^{AK} \right.\right. \\ &\qquad \left.\left. + \sum_{l \in \Psi^S} \left(C_l^{S0} x_{l,t}^{S0} + \sum_{L \in \Psi_l^S} C_l^{SL} x_{l,t}^{SL}\right)\right)\right] \le B. \qquad (17)\end{aligned}$$

The fourth block consists of logical constraints, expressed in terms of the investment and utilization variables, given as follows.

- To avoid more than one change in conductors, only one alteration is permitted for each branch candidate for substitution or addition

$$\sum_{t=1}^{T} \sum_{J \in \Psi_j^R} x_{j,t}^{RJ} \le 1 \quad \{\forall j \in \Psi^R\} \qquad (18)$$

$$\sum_{t=1}^{T} \sum_{K \in \Psi_k^A} x_{k,t}^{AK} \le 1 \quad \{\forall k \in \Psi^A\}. \qquad (19)$$

- Each investment in the substations can be affected, at the most, once

$$\sum_{t=1}^{T} x_{l,t}^{S0} \le 1 \quad \{\forall l \in \Psi^S\} \qquad (20)$$

$$\sum_{t=1}^{T} x_{l,t}^{SL} \le 1 \quad \{\forall l \in \Psi^S, L \in \Psi_l^S\}. \qquad (21)$$

- Investments in an increase in capacity of substations may only be considered after fixed costs have been affected

$$x_{l,t}^{SL} \le \sum_{\tau=1}^{t} x_{l,\tau}^{S} \qquad \begin{matrix}\forall t = 1, \cdots, T\\ \{\forall l \in \Psi^S, L \in \Psi_l^S\}\end{matrix}. \qquad (22)$$

- The fixed network can only be used when it is available

$$0 \le y_{i,t}^{FI} \le y_{\max,i,t}^{FI} \qquad \begin{matrix}\forall t = 1, \cdots, T\\ \{\forall i \in \Psi^F, I \in \Psi_i^F\}\end{matrix}. \qquad (23)$$

- Branches that are candidates for replacement may only be used after the corresponding investment has been made

$$y_{j,t}^{RJ} \le \sum_{\tau=1}^{t} x_{j,\tau}^{RJ} \qquad \begin{matrix}\forall t = 1, \cdots, T\\ \{\forall j \in \Psi^R, J \in \Psi_j^R\}\end{matrix}. \qquad (24)$$

- Investment in any replacement alternative excludes the possibility of using the initial configuration for that branch

$$y_{j,t}^{R0} \leq 1 - \sum_{\tau=1}^{t} \sum_{J \in \Psi_j^R} x_{j,\tau}^{RJ} \qquad \begin{array}{c} \forall t = 1, \cdots, T \\ \{\forall j \in \Psi^R\} \end{array}. \tag{25}$$

- Branches that are candidates for addition may only be used after the relevant investment has been made

$$y_{k,\tau}^{AK} \leq \sum_{t=1}^{\tau} x_{k,t}^{AK} \qquad \begin{array}{c} \forall t = 1, \cdots, T \\ \{\forall k \in \Psi^A, K \in \Psi_k^A\} \end{array}. \tag{26}$$

- Closed paths (meshes) must be avoided

$$\sum_{k \in \Psi^A} \sum_{K \in \Psi_k^A} y_{k,t}^{AK} + \sum_{i \in \Psi^F} \sum_{I \in \Psi_i^F} y_{i,t}^{FI} + \sum_{j \in \Psi^R} \left(y_{j,t}^{R0} + \sum_{J \in \Psi_j^R} y_{j,t}^{RJ} \right) \leq N_t \quad \forall t = 1, \cdots, T. \tag{27}$$

Equation (27) is obtained from the consideration that a radial feeder with $N+1$ nodes (N nodes for the network plus one node for the substation) always leads to a tree having at most N branches [40]. In this way, the inclusion of additional branches would give rise to closed paths in the network. A distribution network containing several radial feeders and substations can be considered as a forest, for which the maximal number of branches is equal to the total number of nodes in the network excluding substation nodes. To ensure that the network obtained is always radial, it may be necessary to add constraints with specific information about the topology of the network under analysis. Considering the diversity of situations encountered, this task can be relatively complex. A simpler strategy was used in this paper which consisted of limiting the total number of active branches at each stage to be less than or equal to the total number of active network nodes at this stage, given by (27). In tests performed so far, the constraint (27), together with additional constraints (new-path and fencing constraints), were sufficient for radial solutions always to be obtained.

C. Additional Constraints

In addition to the aforementioned constraints, a set of additional constraints may be introduced. These additional constraints are based on knowledge of electrical power networks. Two classes of constraints were used in this paper, related to the addition of new paths [38] and to the addition of fencing constraints [41].

1) New-Path Constraints: A new path consists of two or more branches connected in series representing a valid investment and creating a connected path between two or more nodes of a distribution network. During the optimization process, the variables associated with a single path are used independently, without reference to the fact that they are complementary. However, a new path is only created when a suitable combination of branches is used simultaneously since the lack of a single component breaks the series circuit.

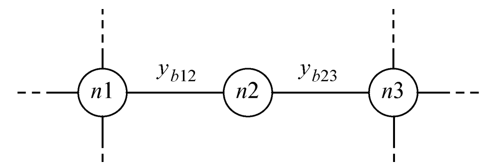

Fig. 2. New path defined for a bridge node with two adjacent branches.

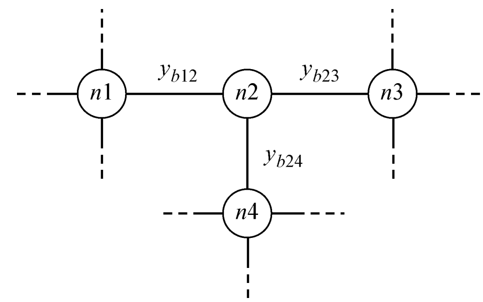

Fig. 3. New path defined for a bridge node with three adjoining circuits.

The way in which constraints on new paths work is illustrated by the simple example in Fig. 2, in which the bridge node $n2$ (without load or generation) has only two adjacent branches, with utilization variables given by y_{b12} and y_{b23}, respectively, for the branches $b12$ and $b23$.

Fig. 2 shows that there is no advantage in separately using the branches $b12$ and $b23$, since the node $n2$ has no load or generation. When these branches are used simultaneously, the nodes $n1 - n2 - n3$ constitute a path. Thus, the alternatives for using this path must respect the following constraint associated with the bridge node $n2$:

$$y_{b12} = y_{b23}. \tag{28}$$

When the bridge node $n2$ has three adjacent branches as shown in Fig. 3, 2^3 combinations exist for utilizing the adjoining branches, of which three combinations are useless (when only one of the branches $b12$, $b23$, or $b24$ is used). The constraints defining acceptable combinations for the network are then

$$y_{b12} \leq y_{b23} + y_{b24} \tag{29.1}$$

$$y_{b23} \leq y_{b12} + y_{b24} \tag{29.2}$$

$$y_{b24} \leq y_{b12} + y_{b23}. \tag{29.3}$$

Introduction of the equality constraint (28) removes two of the four alternatives for using the path shown in Fig. 2 which includes the node $n2$, and is equivalent to removing one binary variable (y_{b12} or y_{b23}). Simultaneously adding the constraints (29) removes three of the eight alternatives for using the path shown in Fig. 3. In the general case, a bridge node with n adjacent branches has 2^n utilization combinations, and n inequality constraints on new paths can be included so as to eliminate n of these alternatives. Although reducing the complexity of the problem of using new-path branches is equal to $n/2^n$, in practice, it is found that n is less than or equal to four branches.

TABLE I
INVESTMENT ALTERNATIVES FOR THE PATH OF FIG. 2

Branch	Investment variable	$f_{\max}$	C
$b12$	x_{b12}^{1}	2	20
	x_{b12}^{2}	3	30
	x_{b12}^{3}	5	50
$b23$	x_{b23}^{1}	1.5	10
	x_{b23}^{2}	2.5	20
	x_{b23}^{3}	4	30

TABLE II
FEASIBLE INVESTMENT ALTERNATIVES FOR THE PATH OF FIG. 2

$f_{\max}$	x_{b12}^{1}	x_{b12}^{2}	x_{b12}^{3}	x_{b23}^{1}	x_{b23}^{2}	x_{b23}^{3}	Cost	Attractive
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**Yes**
0	0	0	0	1	0	0	10	No
0	0	0	0	0	1	0	20	No
0	0	0	0	0	0	1	30	No
0	1	0	0	0	0	0	20	No
1.5	**1**	**0**	**0**	**1**	**0**	**0**	**30**	**Yes**
2	**1**	**0**	**0**	**0**	**1**	**0**	**40**	**Yes**
2	1	0	0	0	0	1	50	No
0	0	1	0	0	0	0	30	No
1.5	0	1	0	1	0	0	40	No
2.5	**0**	**1**	**0**	**0**	**1**	**0**	**50**	**Yes**
3	**0**	**1**	**0**	**0**	**0**	**1**	**60**	**Yes**
0	0	0	1	0	0	0	50	No
1.5	0	0	1	1	0	0	60	No
2.5	0	0	1	0	1	0	70	No
4	**0**	**0**	**1**	**0**	**0**	**1**	**80**	**Yes**

Thus, the benefit from using constraints on new paths is substantial and contributes to reducing the problem size.

For those branches having more than a single alternative, the sum of the utilization variables for each stage t must be used instead of the variables y_{b12}, y_{b23}, and y_{b24}: that is, $\sum_{I\in\Psi_i^F} y_{i,t}^{FI}$ for branches of the fixed network, $\sum_{J\in\Psi_j^R} y_{j,t}^{RJ}$ for branches of the replacement network, and $\sum_{K\in\Psi_k^A} y_{k,t}^{AK}$ for branches of the addition network. This concept can be extended still further for paths having two or more bridge nodes, when it is applied to each bridge node of the path.

Considering the three investment alternatives for branches of the new path in Fig. 2 shown in Table I, there are $2^6 = 64$ combinations for the six binary investment variables. Imposing the constraints (18) and (19) reduces the number of combinations to 4^2, as in each branch, only one investment may be selected.

Considering the investment alternatives in Table I, there are six attractive investment alternatives among the 16 possibilities shown in boldface in Table II. The path capacities ($f_{\max}$), investment variables, and costs of all investment possibilities are also shown in the columns of Table II.

The nonattractive combinations can be eliminated by means of the following constraints:

$$x_{b23}^{1} \leq x_{b12}^{1} \quad (30.1)$$

$$x_{b23}^{2} \leq x_{b12}^{1} + x_{b12}^{2} \quad (30.2)$$

$$x_{b23}^{3} \leq x_{b12}^{2} + x_{b12}^{3} \quad (30.3)$$

$$x_{b12}^{1} + x_{b12}^{2} + x_{b12}^{3} = x_{b23}^{1} + x_{b23}^{2} + x_{b23}^{3}. \quad (30.4)$$

2) Fencing Constraints: Fencing constraints are a generalization of KCL and form part of a heuristic methodology for transmission expansion planning known as the "Fencing Method" [40]. Fig. 4 shows the three kinds of fences used in this paper: 1) around a single node (Type 1); 2) around one node and neighboring node (Type 2); and 3) around a node and its entire neighborhood (Type 3).

When the node $n1$ in Fig. 4 has a current injection (load or generation) different from zero, so that $n1$ must be connected to the network, the type 1 fencing constraint is given by

$$y_{b12} + y_{b14} \geq 1. \quad (31)$$

When there is a nonzero injection to one of the extreme nodes of the branch $b12$ (nodes $n1$ or $n2$), so that at least one of these

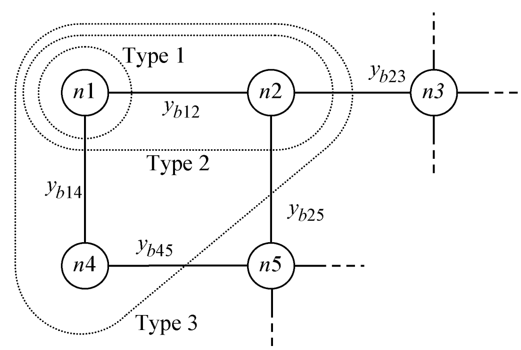

Fig. 4. Fencing constraints around the node $n1$.

extreme nodes of branch $b12$ must be connected to the network, the type 2 fencing constraint is given by

$$y_{b14} + y_{b23} + y_{b25} \geq 1. \quad (32)$$

When there is a nonzero injection at node $n1$ or at one of its neighbors (nodes $n2$ or $n4$), so that at least one of the nodes $n1$, $n2$, $n3$ must be connected to the network, the type 3 fencing constraint is given by

$$y_{b23} + y_{b25} + y_{b45} \geq 1. \quad (33)$$

As noted in the aforementioned discussion on new path constraints, when branches have more than one investment alternative, instead of using the variables y_{b12}, y_{b14}, y_{b23}, y_{b25}, and y_{b45}, the sum of the utilization variables must be used at each stage.

III. LOAD AND NETWORK REPRESENTATIONS

The load and network model used in this paper is adapted from the dc load-flow linearized network model given in [37]. In the dc load-flow model [37], power injections, phase angles of nodal voltages, and branch reactances are used; instead, the formulation presented here uses current injections, magnitudes of nodal voltages, and the absolute value of branch impedances.

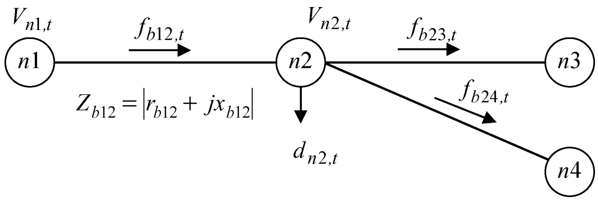

Fig. 5. Part of a network.

The voltage drop in a branch is given as the product of the absolute values of the impedance and the current flow. For the branch $b12$ shown in Fig. 5, the following expressions (in per unit) are obtained from the KCL and the KVL, respectively:

$$f_{b12,t} = d_{n2,t} + f_{b23,t} + f_{b24,t} \tag{34}$$

$$\Delta V_{b12,t} = V_{n1,t} - V_{n2,t} = Z_{b12} f_{b12,t}. \tag{35}$$

The constraint (2) is obtained from (34); the constraints (3)–(6) are obtained from (35). In the networks that have been analyzed, mean errors in the magnitudes of nodal voltages obtained using this approximate model were adequate for the study objectives, when compared with the solution for load flow. In addition, it becomes possible to relate voltage magnitudes to current flows by means of a linear equation. Thus, in the proposed load and network model, the inclusion of constraints on voltage drop does not increase the complexity of the optimization model. If the classical (ac) load-flow model had been used, the optimization problem would have to handle nonlinear equations making the solution extremely difficult.

IV. Conclusion

The multistage optimization model presented in this paper proved to be wide ranging and flexible. It made it possible to consider the most common alteration to a distribution network: 1) change in the cross section of already-existing conductors (replacement); 2) insertion of new branches with different cross-sections (addition); 3) installation of new substations; 4) increasing the capacity of existing substations; 5) use of available capacity for distributed generation; and 6) long-term planning horizon.

The simplification used in representing KVL allowed an optimization model to be formulated in which all constraints were linear, by means of the linear disjunctive model. The resulting linear model was solved by means of optimization software based on the branch-and-bound method to obtain the best solution. Although the problem may have a large number of binary variables, the introduction of logical constraints (18)–(27) and of additional constraints (28)–(33) significantly reduced the search space, ensuring that the mixed integer problem was computationally tractable.

References

[1] H. L. Willis, *Power Distribution Planning Reference Book*, 2nd ed. New York: Marcel Dekker, 2004, p. 1217.

[2] S. K. Khator and L. C. Leung, "Power distribution planning: A review of models and issues," *IEEE Trans. Power Syst.*, vol. 12, no. 3, pp. 1151–1159, Aug. 1997.

[3] E. Lakervi and E. J. Holmes, *Electricity Distribution Network Design*, 2nd ed. London, U.K.: IEE Power Ser. 21, 1995, p. 325.

[4] H. K. Temraz and V. H. Quintana, "Distribution system expansion planning models: An overview," *Elect. Power Syst. Res.*, vol. 26, pp. 61–70, 1993.

[5] U. G. Knight, "The logical design of electrical networks using linear programming methods," *Proc. Inst. Elect. Eng.*, vol. 33, pp. 306–314, 1960.

[6] D. I. Sun, D. R. Farris, P. J. Cote, R. R. Shoults, and M. S. Chen, "Optimal distribution substation and primary feeder planning via the fixed charge network formulation," *IEEE Trans. Power App. Syst.*, vol. PAS-101, no. 3, pp. 602–609, Mar. 1982.

[7] M. A. El-Kady, "Computer-aided planning of distribution substation and primary feeders," *IEEE Trans. Power App. Syst.*, vol. PAS-103, no. 6, pp. 1183–1189, Jun. 1984.

[8] K. Aoki, K. Nara, T. Satoh, M. Kitagawa, and K. Yamanaka, "New approximate optimization method for distribution system planning," *IEEE Trans. Power Syst.*, vol. 5, no. 1, pp. 126–132, Feb. 1990.

[9] T. Gönen and B. L. Foote, "Mathematical dynamic optimization model for electrical distribution system planning," *Elect. Power Energy Syst.*, vol. 4, no. 2, pp. 129–136, Apr. 1982.

[10] I. J. Ramírez-Rosado and T. Gönen, "Pseudodynamic planning for expansion of power distribution systems," *IEEE Trans. Power Syst.*, vol. 6, no. 1, pp. 245–254, Feb. 1991.

[11] K. Nara, T. Satoh, K. Aoki, and M. Kitagawa, "Multi-year expansion planning for distribution systems," *IEEE Trans. Power Syst.*, vol. 6, no. 3, pp. 952–958, Aug. 1991.

[12] K. Nara, T. Satoh, H. Kuwabara, K. Aoki, M. Kitagawa, and T. Ishihara, "Distribution systems expansion planning by multi-stage branch exchange," *IEEE Trans. Power Syst.*, vol. 7, no. 1, pp. 208–214, Feb. 1992.

[13] H. Kuwabara and K. Nara, "Multi-year and multi-state distribution systems expansion planning by multi-stage branch exchange," *IEEE Trans. Power Del.*, vol. 12, no. 1, pp. 457–463, Jan. 1997.

[14] M. Vaziri, K. Tomsovic, and A. Bose, "A direct graph formulation of the multistage distribution expansion problem," *IEEE Trans. Power Del.*, vol. 19, no. 3, pp. 1335–1341, Jul. 2004.

[15] M. Vaziri, K. Tomsovic, and A. Bose, "Numerical analyses of a direct graph formulation of the multistage distribution expansion problem," *IEEE Trans. Power Del.*, vol. 19, no. 3, pp. 1348–1354, Jul. 2004.

[16] P. C. Paiva, H. M. Khodr, J. A. Domínguez-Navarro, J. M. Yusta, and A. J. Urdaneta, "Integral planning of primary-secondary distribution systems using mixed integer linear programming," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 1134–1143, May 2005.

[17] M. Ponnavaikko, K. S. P. Rao, and S. S. Venkata, "Distribution system planning through a quadratic mixed integer programming approach," *IEEE Trans. Power Del.*, vol. PWRD-2, no. 4, pp. 1157–1163, Oct. 1987.

[18] H. K. Youssef and R. Hackam, "Dynamic solution of distribution planning in intermediate time range," *IEEE Trans. Power Del.*, vol. 3, no. 1, pp. 341–348, Jan. 1988.

[19] W. El-Khattam, Y. G. Hegazy, and M. M. A. Salama, "An integrated distributed generation optimization model for distribution system planning," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 1158–1165, May 2005.

[20] N. G. Boulaxis and M. P. Papadopoulos, "Optimal feeder routing in distribution system planning using dynamic programming technique and GIS facilities," *IEEE Trans. Power Del.*, vol. 17, no. 1, pp. 242–247, Jan. 2002.

[21] E. Díaz-Dorado and J. C. Pidre, "Optimal planning of unbalanced networks using dynamic programming optimization," *IEEE Trans. Power Syst.*, vol. 19, no. 4, pp. 2077–2085, Nov. 2004.

[22] M. A. Farrag, M. M. El-Metwally, and M. S. El-Bages, "A new model for distribution system planning," *Elect. Power Energy Syst.*, vol. 21, pp. 523–531, 1999.

[23] M. Vaziri, K. Tomsovic, and T. Gönen, "Distribution expansion problem revisited: Part 2—Proposed modeling and formulation," in *Proc. 4th Int. Conf. Power Energy Systems*, Marbella, Spain, Sep. 19–22, 2000, pp. 283–290.

[24] K. Nara, Y. Hayashi, S. Muto, and K. Tuchida, "A new algorithm for distribution feeder expansion planning in urban area," *Elect. Power Syst. Res.*, vol. 46, pp. 185–193, 1998.

[25] E. Míguez, J. Cidrás, E. Díaz-Dorado, and J. L. García-Dornelas, "An improved branch-exchange algorithm for large-scale distribution network planning," *IEEE Trans. Power Syst.*, vol. 17, no. 4, pp. 931–936, Nov. 2002.

[26] V. Miranda, J. V. Ranito, and L. M. Proença, "Genetic algorithms in optimal multistage distribution network planning," *IEEE Trans. Power Syst.*, vol. 9, no. 4, pp. 1927–1933, Nov. 1994.

[27] I. J. Ramírez-Rosado and J. L. Bernal-Augustín, "Genetic algorithms applied to the design of large power distribution systems," *IEEE Trans. Power Syst.*, vol. 13, no. 2, pp. 696–703, May 1998.

[28] I. J. Ramírez-Rosado and J. L. Bernal-Augustín, "Reliability and costs optimization for distribution networks expansion using an evolutionary algorithm," *IEEE Trans. Power Syst.*, vol. 16, no. 1, pp. 111–118, Feb. 2001.

[29] E. Díaz-Dorado, J. Cidrás, and E. Míguez, "Application of evolutionary algorithms for the planning of urban distribution networks of medium voltage," *IEEE Trans. Power Syst.*, vol. 17, no. 3, pp. 1151–1159, Aug. 2002.

[30] Y. Y. Hsu and J. L. Chen, "Distribution planning using a knowledge based expert systems," *IEEE Trans. Power Del.*, vol. 5, no. 3, pp. 1514–1519, Jul. 1990.

[31] R. Ranjan, B. Venkatesh, and D. Das, "A new algorithm for power distribution system planning," *Elect. Power Syst. Res.*, vol. 62, no. 1, pp. 55–65, 2002.

[32] J. F. Gómez, H. M. Khodr, P. M. de Oliveira, L. Ocque, J. M. Yusta, R. Villasana, and A. J. Urdaneta, "Ant colony system algorithm for the planning of primary distribution circuits," *IEEE Trans. Power Syst.*, vol. 19, no. 2, pp. 996–1004, May 2004.

[33] V. Parada, J. A. Ferland, M. Arias, and K. Daniels, "Optimization of electrical distribution feeders using simulated annealing," *IEEE Trans. Power Del.*, vol. 19, no. 3, pp. 1135–1141, Jul. 2004.

[34] I. J. Ramírez-Rosado, J. A. Domínguez-Navarro, and J. M. Yusta-Loyo, "A new model for optimal electricity distribution planning based on fuzzy set techniques," in *Proc. IEEE Power Eng. Soc. Summer Meeting*, Jul. 1999, vol. 2, pp. 1048–1054.

[35] A. Augugliaro, L. Dusonchet, and E. R. Sanseverino, "An evolutionary parallel tabu search approach for distribution systems reinforcement planning," *Adv. Eng. Inform.*, no. 16, pp. 205–215, 2002.

[36] L. Bahiense, G. C. Oliveira, M. Pereira, and S. Granville, "A mixed integer disjunctive model for transmission network expansion," *IEEE Trans. Power Syst.*, vol. 16, no. 3, pp. 560–565, Aug. 2001.

[37] R. Romero, A. Monticelli, A. Garcia, and S. Haffner, "Test systems and mathematical models for transmission network expansion planning," *Proc. Inst. Elect. Eng., Gen. Transm Distrib.*, vol. 1491, no. 1, pp. 27–36, 2002.

[38] A. Monticelli, A. Santos, M. V. F. Pereira, S. Cunha, J. G. Praça, and B. Park, "Interactive transmission network planning using least-effort criterion," *IEEE Trans. Power App. Syst.*, vol. PAS-101, no. 10, pp. 3919–3925, Oct. 1982.

[39] S. Haffner, L. F. A. Pereira, L. A. Pereira, and L. S. Barreto, "Multistage model for distribution expansion planning with distributed generation—Part II: Numerical results," *IEEE Trans. Power Del.*, vol. 23, no. 2, pp. 924–929, Apr. 2008.

[40] R. K. Ahuja, T. L. Magnati, and J. B. Orlin*, Network Flows: Theory, Algorithms and Applications*. Upper Saddle River, NJ: Prentice-Hall, 1993, p. 846.

[41] C. J. Baldwin, C. A. DeSalvo, C. H. Hoffman, and W. S. Ku, "A model for transmission planning by logic," *AIEE Trans. Power App. Syst.*, vol. PAS-78, pp. 1638–1645, 1960.

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