

COMPARISON OF LINEAR QUADRATIC CONTROLLERS WITH STABILITY ANALYSIS FOR DC-DC BOOST CONVERTERS UNDER LARGE LOAD RANGE

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ABSTRACT

This paper presents four different control design strategies for linear quadratic regulators applied to DC-DC boost converters operating under large load range, pointing out advantages and drawbacks by means of a performance comparison. Two of the strategies are based on single controllers and the other two use a set of local controllers to cope with the entire load range. The robust stability of the closed-loop linearized system with the four control strategies is investigated via the existence of a common Lyapunov function, obtained in a fast and efficient way by the solution of linear matrix inequality conditions.

Key Words: Boost converter, fuzzy, genetic algorithms, linear quadratic regulator, lyapunov stability, linear matrix inequalities.

I. INTRODUCTION

Boost converters are used in several important applications, such as power factor correction devices, telecommunication systems, electric motor drivers and, more recently, in renewable energy sources [1–3]. One problem of interest concerning the boost converter is to keep stability and good regulation of the output voltage for operation under large load variation [4, 5]. Since the model of the converter is nonlinear [6–8], controllers designed for a nominal load may perform poorly when the converter switches to another load, for instance. Additionally, classical control strategies designed for a nominal operating point usually do not provide a mathematical proof of robust stability, only assessing the closed-loop stability by means of circuit simulation for chosen disturbances or evaluate the local stability by means of linear techniques as frequency response analysis [1]. These strategies cannot cover a set of infinite values for the load parameter in their evaluation and also cannot cope with arbitrarily fast parameter variations, leading to the interest other proofs of stability.

In the context of in boost converters, one has the sliding-mode control as a very often used technique (see, for instance [4, 9–11] and references therein). Although global stability, performance, and robustness to parametric uncertainty can be achieved for the entire space of operation, one drawback for nonlinear control techniques as sliding-mode is the lack of a systematic and general procedure for control design. A way to overcome this difficulty in using nonlinear techniques, also applicable for power converters, is to apply fuzzy logic based controllers [5, 12–14], which allow the control designer to inform the decision variables and the fuzzy rules from which the control signal will be synthesized. A limitation with this procedure is to find a formal proof of stability for the closed-loop system.

Linear techniques are very well known, with general control design tools, and can be directly applied to control synthesis for the linearized model of the boost converter, under the assumption that the converter suffers small perturbation around some equilibrium point [1]. In this area, state feedback techniques are very important, specially in optimal control. Here one is concerned with the optimal linear quadratic regulator (LQR), which is able to minimize the energy of the control signal, and leads to closed-loop phase margins greater than 60 degrees and infinite gain margins, appropriate features for power converters [15–17]. However, the conventional LQR does not take into account parametric uncertainty in the control design. To overcome this problem, a robust LQR was given in [18], but still suffers from the problem of choosing the weighting matrices of the cost function heuristically. To help the control designer with the choice of such matrices, one can use genetic algorithms (GA), whose objective function can be chosen in order to optimize the system performance [19–21]. In order to do this, works such as

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[20, 21] use the results of computer simulation of a linear mathematical model to evaluate the objective function, which might not be an appropriate way to get this function for the case of the boost converter (nonlinear system) which is subject to sudden changes on the load.

Differently from [18] and other similar robust control design techniques based on quadratic stability, here one uses a GA based on data from circuit simulation to get suitable LQR controllers without long trial and errors phases for the control designer. Then, *a posteriori*, linear matrix inequalities (LMIs, [22]) are used to certify the stability of the linearized plant with the controllers obtained with the GA. This procedure tends to produce less conservative controllers with better performances, even when compared with robust controllers with a single gain, as shown in Section VII. It is also worth citing the recent work [23], where the authors derived LMI conditions to cope with uncertainties, disturbances and bilinearities present in boost converters, synthesizing a control law which ensures stability for a domain of initial values and operating conditions.

This paper is concerned with four different control design strategies based on LQR for boost with large load range. The first strategy is based on a single LQR, designed for the nominal load condition with the choice of the weighting matrices based on heuristics. The second strategy is similar to the first but employs a GA to help in the choice of the LQR weighting matrices, allowing improvement of the performance for the nominal load condition but also suffering from operation in large load range. This GA is specialized to optimize the closed-loop performance to load disturbances based on data obtained from an electrical circuit simulator, this being a contribution of this work. The third and fourth strategies employ a set of local LQRs designed with the help of the proposed GA, but the third strategy uses direct switching between the local controllers and the fourth uses a fuzzy logic combination of these controllers. The comparative analysis indicates that the fuzzy logic based strategy has a superior performance over the entire load range, even when compared with the results obtained using the robust controller from [18]. As a final contribution, this paper provides a mathematical analysis of stability of the closed-loop system for load values belonging to the range under consideration and with any of the controllers investigated allows conclusions to be drawn about the stability based on the existence of a common Lyapunov function.

II. PROBLEM FORMULATION

The DC-DC boost converter is the plant being considered here. The electrical circuit model of the plant, including parasitic resistances, is given in Fig. 1. For continuous conduction mode (CCM) operation [1], this system

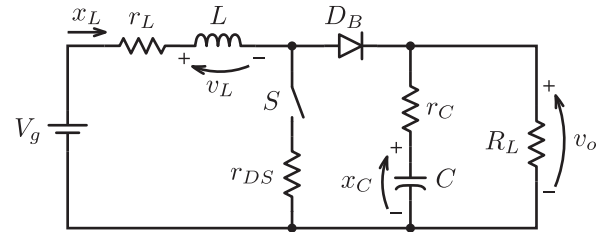


Fig. 1. Circuit of the boost converter with losses.

can be described in the state-space as a switched linear system [24]:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}_i \mathbf{x} + \mathbf{B}_i V_g \\ v_o = \mathbf{C}_i \mathbf{x} \end{cases} \quad i = 1, 2 \quad (1)$$

with two subsystems whose descriptions depend on the state of the switch S. Choosing the state vector as $\mathbf{x} = [x_L \ x_C]^T$, where the superscript^T denotes transpose, the model of the subsystem concerning the switch on is given by the matrices

$$\begin{aligned} \mathbf{A}_1 &= \begin{bmatrix} -\frac{r_L + r_{DS}}{L} & 0 \\ 0 & -\frac{1}{C(R_L + r_C)} \end{bmatrix} \\ \mathbf{B}_1 &= \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \quad \mathbf{C}_1 = \begin{bmatrix} 0 & \frac{R_L}{R_L + r_C} \end{bmatrix} \end{aligned} \quad (2)$$

and the subsystem concerning the switch off is given by

$$\begin{aligned} \mathbf{A}_2 &= \begin{bmatrix} -\frac{R_L(r_L + r_C) + r_L r_C}{L(R_L + r_C)} & -\frac{R_L}{L(R_L + r_C)} \\ \frac{R_L}{C(R_L + r_C)} & -\frac{1}{C(R_L + r_C)} \end{bmatrix} \\ \mathbf{B}_2 &= \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \quad \mathbf{C}_2 = \begin{bmatrix} \frac{R_L r_C}{R_L + r_C} & \frac{R_L}{R_L + r_C} \end{bmatrix}. \end{aligned} \quad (3)$$

From (1)–(3) one can obtain the average model [1]

$$\begin{cases} \dot{\bar{\mathbf{x}}} = \mathbf{A} \bar{\mathbf{x}} + \mathbf{B} V_g \\ \bar{v}_o = \mathbf{C} \bar{\mathbf{x}} \end{cases} \quad (4)$$

whose matrices are given by

$$\mathbf{A} = \mathbf{A}_1 D + \mathbf{A}_2 D' \quad (5)$$

$$\mathbf{B} = \mathbf{B}_1 D + \mathbf{B}_2 D' \quad (6)$$

$$\mathbf{C} = \mathbf{C}_1 D + \mathbf{C}_2 D' \quad (7)$$

with being D the nominal duty cycle and $D' = 1 - D$. The equilibrium point is obtained by means of

$$\mathbf{X}_q = -\mathbf{A}^{-1}\mathbf{B}V_g. \quad (8)$$

Assuming the converter operates around a given equilibrium point and suffers only small perturbations, a linearized state-space model of the plant can be given by [1, 25]

$$\begin{cases} \dot{\tilde{\mathbf{x}}} = \mathbf{F}\tilde{\mathbf{x}} + \mathbf{G}\tilde{d} \\ \tilde{v}_o = \mathbf{H}\tilde{\mathbf{x}} + \mathbf{E}\tilde{d} \end{cases} \quad (9)$$

where is $\tilde{d}$ a small perturbation introduced in the control signal, and the systems matrices are written as

$$\mathbf{F} = \mathbf{A} \quad (10)$$

$$\mathbf{G} = (\mathbf{A}_1 - \mathbf{A}_2)\mathbf{X}_q \quad (11)$$

$$\mathbf{H} = \mathbf{C} \quad (12)$$

$$\mathbf{E} = (\mathbf{C}_1 - \mathbf{C}_2)\mathbf{X}_q \quad (13)$$

Notice that the model (9) is suitable for state feedback control techniques. These techniques are especially adequate to deal with several optimal controllers, and one important optimal control, the LQR, will be used in this paper [15-17, 26].

An integral action over the tracking error is included in the system model by means of an extra state variable x_e in order to ensure zero steady-state error for a constant reference signal. This extra state is given by

$$x_e(t) \triangleq \int_0^t e(\tau) d\tau = \int_0^t -\tilde{v}_o(\tau) d\tau \quad (14)$$

leading to the dynamical equation

$$\dot{x}_e = -\mathbf{H}\tilde{\mathbf{x}} - \mathbf{E}\tilde{d}. \quad (15)$$

Including this equation in the model of the system, one has the following augmented representation

$$\dot{\xi} = \hat{\mathbf{F}}\xi + \hat{\mathbf{G}}\tilde{d} \quad (16)$$

being

$$\xi = \begin{bmatrix} \tilde{\mathbf{x}} \\ x_e \end{bmatrix}, \quad \hat{\mathbf{F}} = \begin{bmatrix} \mathbf{F} & \mathbf{0} \\ -\mathbf{H} & 0 \end{bmatrix}, \quad \hat{\mathbf{G}} = \begin{bmatrix} \mathbf{G} \\ -\mathbf{E} \end{bmatrix} \quad (17)$$

Since the focus of this paper is to apply LQR control schemes and that this is a state feedback control law given by

$$\tilde{d} = -\mathbf{K}\xi \quad (18)$$

the control design problem will be to determine the entries of the gain vector $\mathbf{K}$, given by [16, 27]

$$\mathbf{K} = [K_i \quad K_v \quad K_e]. \quad (19)$$

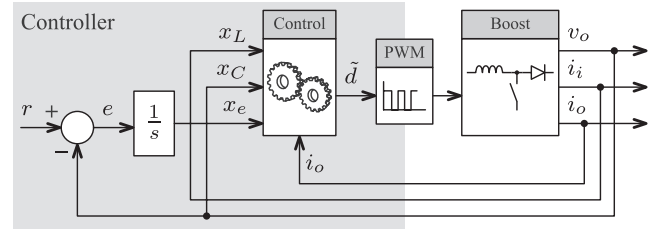


Fig. 2. General block diagram for testing the controllers.

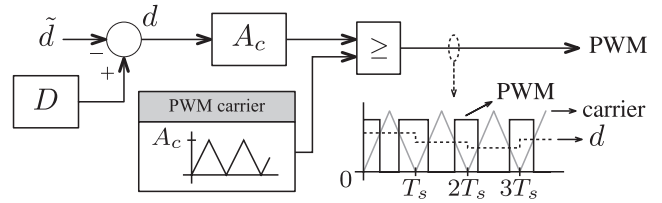


Fig. 3. Block diagram of the pulse width modulator.

The application of this state feedback control law can be seen in Fig. 2, where the control gains belong to the control block. The aim now is to define four control design problems based on LQR in order to get an appropriate control block in Fig. 2. The pulse width modulator (PWM) block is implemented as the block diagram shown by Fig. 3, where A_c is the amplitude of the triangular carrier.

Problem 1. Given an equilibrium point, design a LQR using the conventional method to ensure stability in this point and evaluate the performance of the system to large load variations.

Problem 2. Improve the performance of the controller designed in Problem 1 with the help a GA.

Problem 3. Given a range of loads for the boost converter, design a family of LQRs with the aid of GA to different operation points and a strategy of switching between these controllers to ensure good dynamic performance.

Problem 4. Considering the same load range and the same controllers as Problem 3, employ a fuzzy supervisor to carry out a convex combination of the controllers aiming to improve the robustness to load changes for the closed-loop system.

To evaluate the strategies presented throughout the paper, a converter of 1.5 kW is employed, whose specifications are shown in Table I. With these values, the converter will operate at CCM for the entire load range evaluated in this work.

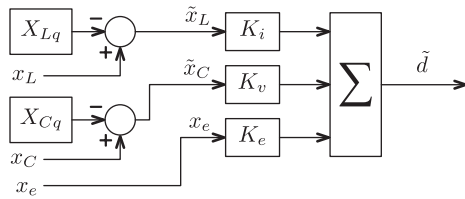


Fig. 4. Block diagram of the controller.

Table I. Parameters of the boost converter.

Parameter	Value	Parameter	Value
V_g	56 V	L	602.11 μ H
V_o	200 V	r_L	5 m Ω
F_s	50 kHz	C	27 μ F
D	0.72	r_C	50 m Ω
R_L	26.66 Ω	r_{DS}	10 m Ω

Solutions for Problems 1 to 4 are given in the following, detailing the control structures, the computation of the control gains and the results in each case, with comparisons of the results.

III. SOLUTION OF PROBLEM 1

The control structure to solve Problem 1 is given in Fig. 4. Notice that the output current variable i_o is not used in this strategy. The aim of the control design now is to compute the gains K_i , K_v , and K_e optimizing the quadratic cost function from the LQR

$$J_{LQR} = \int_0^\infty (\xi^T Q \xi + \tilde{d} R \tilde{d}) dt \quad (20)$$

where matrix Q is symmetric positive semidefinite and R is a positive real scalar. The control gains that minimize the cost function in (20) are given by the Riccati equation [16], and its solution can be easily obtained by means of specialized programs such as MATLAB (lqr function). The LQR is a very popular optimal controller that can provide good stability margins [17, 18, 26].

However, the design of an LQR is strongly based on the appropriate choice of Q and R , that depends on the knowledge of the control system and usually on trial and error design sessions [21].

Consider 100% of output power as the nominal load condition, for which the operating point is

$$\begin{cases} x_{L,q,100} = 26.503 \text{ A} \\ x_{C,q,100} = 197.9 \text{ V.} \end{cases} \quad (21)$$

Based on the parameters of Table I and on (21), one has the matrices $\hat{F}$ and $\hat{G}$ to be used in the LQR design. It is worth

mentioning that this pair is controllable. As conventionally done, the weight values of Q and R are chosen heuristically, and a good choice was

$$Q_{100} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \times 10^6 \end{bmatrix} \quad (22)$$

$$R_{100} = 10^4 \quad (23)$$

producing the control gain vector

$$K_{LQR,100} = [0.0467925 \quad 0.0029557 \quad -10]. \quad (24)$$

It can be expected that this controller ensures stability around the equilibrium point, and that the closed-loop performance may become poorer as the circuit operates far from the nominal load condition. This will be confirmed in Section VII, where a detailed comparison between the solution of problems 1 to 4 is provided.

It is worth mentioning that the choice of the weights for Q and R with structure in (22) was made by trial and error in a space where four weight values are positive and real. This means an unbounded space of search which was investigated only for positive integers, which is common in LQR design. This space of search can be investigated with better choices of Q and R with the help of a GA as follows.

IV. SOLUTION OF PROBLEM 2

The GA is a technique of search and optimization based on the natural selection principle, being one of the most known evolutionary computation techniques [19, 21]. The algorithm starts with a random population of individuals (chromosomes), and through genetic processes similar to those occurring in nature, evolve under specified rules in order to minimize an objective function. Since the population is generated randomly, the GA is able to virtually search the entire solution space, and provides simultaneous searches at different points in this space. During the algorithm execution, the chromosomes that present the best characteristics (lowest cost) generate offspring, improving the average cost value of the population as a whole.

Due to the quantity of parameters and the stochastic nature of the process, the algorithm can converge to different results each run or be confined in a local minimum point. To mitigate this problem, sufficiently large population values, and different rates of mutation and crossover can be used to ensure a good population diversity. Moreover, elitism ensures the survival of the best chromosomes for the next generation.

To help the LQR design, the GA objective is now to determine Q and R so that the closed-loop response presents a good tradeoff between overshoot and integral of time

Table II. Parameters of genes represented by chromosome.

Gene	1	2	3	4
Parameter	q_{11}	q_{22}	q_{33}	r_{11}

squared error (ITSE) index [16] in the event of a load disturbance. Define each chromosome with the genetic structure in Table II, which corresponds to weighting matrices with entries given by

$$\mathbf{Q}_{GA} = \begin{bmatrix} q_{11} & 0 & 0 \\ 0 & q_{22} & 0 \\ 0 & 0 & q_{33} \end{bmatrix} \quad (25)$$

$$R_{GA} = r_{11}. \quad (26)$$

The objective function to be minimized by the GA is defined by

$$J_{GA} = \int_{t_b}^{t_e} e(\tau)^2 d\tau \int_{t_b}^{t_e} |d(\tau)| \tau d\tau \quad (27)$$

where e is the error with respect to the reference, d is the control action, t_b and t_e are the times of beginning and end of the transient caused by the load disturbance, respectively. These times are given by the control designer based on the knowledge of the system response.

Notice the GA here uses data from the responses of the closed-loop system obtained by means of specialized circuit simulators to evaluate its objective function, which is different from most similar works with GA helping the design of controllers, for which the GA objective function is based on mathematical metrics obtained from the system model. The GA plays the role of the control designer in Problem 1, being here an automatic control designer that performs its task following the steps given by:

- (i) Randomly creates the initial population;
- (ii) Determines the LQR gains for each chromosome using MATLAB;
- (iii) Simulates the closed-loop system in SIMULINK with the help of Matlab and Simulink for simulation of the power circuit for each set of LQR gains, reducing the resistive load by 25% at 20 ms;
- (iv) Calculates the objective function (27) for each simulation result;
- (v) Sort results based on their fitness values. If the stop condition is met, skip to step vii;
- (vi) Creates the next generation from changes in the chromosome of a single parent (mutation) or combination of genes from a pair of chromosomes (crossover). Return to step ii;
- (vii) Determines the best weighting matrices for the LQR based on the best chromosome;

Table III. Best chromosome found for the LQR for the nominal load condition.

Gene	1	2	3	4
Value	0.167076	0.015611	3.4148×10^6	347031.2

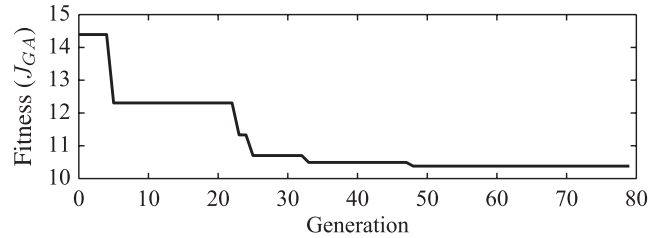


Fig. 5. Evolution of the fitness of the best chromosome for the controller designed to 100% of load.

(viii) Terminates the algorithm.

In the solution of the GA, one uses here MATLAB. The population is kept in 60 chromosomes, each gene limited to the range $[0 \ 5 \times 10^6]$, which is a large real interval of search in opposition to the search performed only for positive integers in Problem 1. The two best chromosomes of each generation have ensured its perpetuation (elitism). Of the remainder, excluding the elitism, each new generation is composed of 30% of crossover between pairs of chromosomes and the other 60% are generated by mutations. After 79 generations the best chromosome found was at presented in Table III, leading to the gain vector defined in

$$\mathbf{K}_{GA,100} = [8.083 \times 10^{-3} \quad 416.1 \times 10^{-6} \quad -3.137] \quad (28)$$

The evolution of the fitness of the best chromosome is shown in Fig. 5. It is worthy of mention that the computation time of the algorithm is 17 min, considering the simulation of the switched converter. Notice the difference between the weights from Table III and the weights in (22). The weights from Table III would be difficult to find by trial and error or by exhaustive gridding procedures.

The control structure in Fig. 4 is used here, since the strategy is implemented in the same way as the one from Problem 1. However, it is expected that the controller designed with the help of the GA can provide a better closed-loop performance than the one obtained from the solution of Problem 1, as will be illustrated Section VII.

V. SOLUTION OF PROBLEM 3

As can be observed by (2), (3), and (8), the variation of load resistance directly changes the dynamics and also the equilibrium point of the system. It can be expected that several controllers designed to work in specific ranges of load,

Table IV. Equilibrium points of the model for different output power.

P_o (%)	$X_{L,q}$ (A)	$X_{C,q}$ (V)	R_L (Ω)	I_o (A)
100	26.503	197.89	26.666	7.421
75	19.930	198.41	35.555	5.580
50	13.321	198.94	53.333	3.730
25	6.678	199.46	106.666	1.870

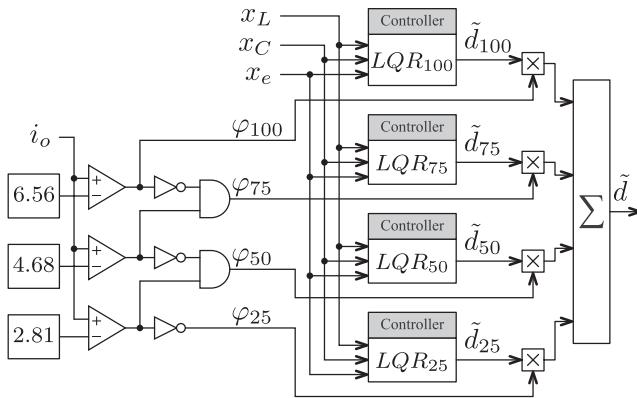


Fig. 6. Block diagram to switching controllers.

when properly selected or combined, will ensure a better performance than a single controller designed for a single operating point but coping with the entire load range, as those from Problems 1 and 2.

Consider now the load range from 25% to 100% of the output power, and four representative load situations in this range, leading to the equilibrium points and the steady-state output currents given in Table IV. These four load situations will be used to design optimal local controllers. Each local LQR controller is computed with the help of the GA developed in Problem 2, leading to

$$\mathbf{K}_{GA,75} = [9.468 \times 10^{-3} \quad 527.38 \times 10^{-6} \quad -2.907] \quad (29)$$

$$\mathbf{K}_{GA,50} = [10.457 \times 10^{-3} \quad 1.024 \times 10^{-3} \quad -3.760] \quad (30)$$

$$\mathbf{K}_{GA,25} = [21.417 \times 10^{-3} \quad 4.936 \times 10^{-3} \quad -10.57]. \quad (31)$$

Notice that $\mathbf{K}_{GA,25}$ is the LQR gain vector for the load situation of 25% of the output power and $\mathbf{K}_{GA,100}$ is the gain (28), solution of Problem 2.

The most appropriate controller with respect to the load situation will be selected based on the structure in Fig. 6. The output current i_o , which is related to the load situation is used as the decision variable, as shown in Fig. 7. The thresholds, given in Table V, are chosen here to determine the limits to decide the switching from one local controller to another.

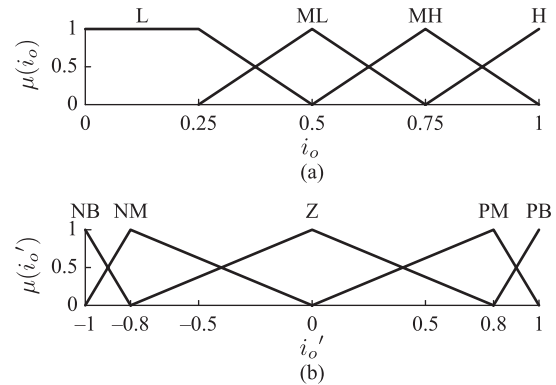


Fig. 7. Membership functions of the fuzzy supervisor control: (a) output current, and (b) change in output current.

Table V. Thresholds for switching controllers.

Output power (%)	Load current (A)
87.5	6.562
62.5	4.687
37.5	2.812

Notice from Fig. 6 that the control signal is obtained by means of a combination of the control signals of each local controller, being the combination given by

$$\tilde{d} = \varphi_{100}\tilde{d}_{100} + \varphi_{75}\tilde{d}_{75} + \varphi_{50}\tilde{d}_{50} + \varphi_{25}\tilde{d}_{25} \quad (32)$$

where, for instance, $\tilde{d}_{25} = -\mathbf{K}_{GA,25}\xi$.

In (32) one has a special case of a convex combination of signals of the four controllers, when only one of the weights φ_{100} , φ_{75} , φ_{50} , and φ_{25} is equal to 1, indicating the active controller for the load situation, and the others are equal to 0.

One can expect from this strategy that the output of the closed-loop system will be better than the one obtained from the single controller strategies of Problems 1 and 2, since one has specialized controllers for each load situation. As commonly happens in switching control strategies, one may have non smoothness in the control signal when switching the control gains and also problems of control switching near the threshold values. These facts will be confirmed in Section VII.

VI. SOLUTION OF PROBLEM 4

One of the problems of switching control strategies is the discontinuity of the control signal caused by the sudden transition between different sets of gains. This problem can be mitigated if the control weights in (32) can assume values between 0 and 1 and not only 0 or 1. This can be given when the signals φ_{100} , φ_{75} , φ_{50} , and φ_{25} are computed from a

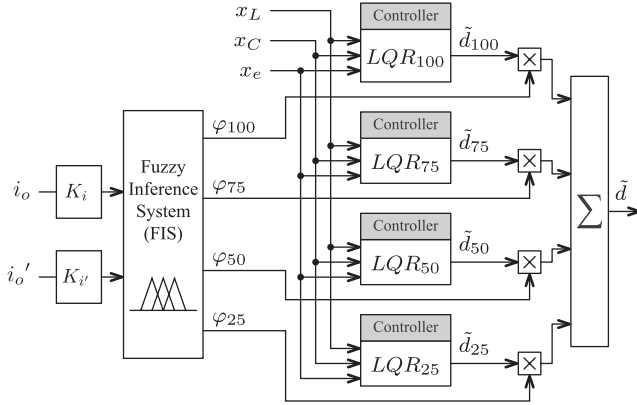


Fig. 8. Block diagram for combining controllers.

fuzzy logic based supervisor. Notice this control strategy is similar to the one in Problem 3 with the same local controllers, but combining them instead of selecting only one active controller, as given in Fig. 8. The supervisor control uses two input variables. The first one, the output current (i_o) is divided into four membership functions denominated L (low), ML (medium low), MH (medium high) and H (high), which corresponds to the four ranges of the output power used for each LQR—25%, 50%, 75% and 100% of the output power, respectively. The second one, the derivative of the output current (i_o'), is divided into five membership functions, denominated NB (negative big), NM (negative medium), Z (zero), PM (positive medium) and PB (positive big). The i_o' variable acts to anticipate the choice of most appropriate controller in the occurrence of significant load variations. Before being used by the inference process, the signals i_o and i_o' are scaled, to adjust them to the variable universe of discourse. Such scaling gains are defined by

$$\begin{cases} K_i = 0.1333 \\ K_{i'} = 16 \times 10^{-6} \end{cases} \quad (33)$$

where being K_i is just a normalization factor for the maximum output current, and $K_{i'}$ is determined by trial and error.

The inference process of Takagi-Sugeno has been used here. The defuzzification method is the weighted average of the rules activation, expressed in

$$\varphi_\delta = \frac{\sum_{j=1}^n w_j z_j}{\sum_{j=1}^n w_j} \quad (34)$$

where δ represents one of the four outputs of the fuzzy supervisor, z_j is a constant obtained from the rule base and w_j is the value of the activation rule, given by

$$w_i = \text{AND}(\mu(i_o), \mu(i_o')) \quad (35)$$

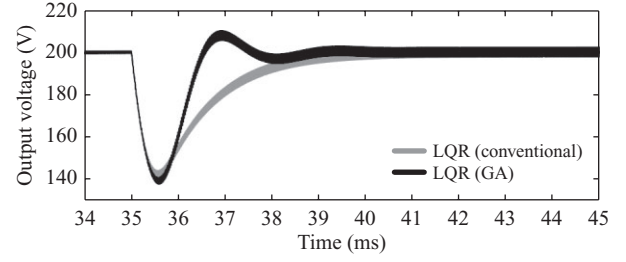


Fig. 9. Comparison between the conventional LQR and the LQR obtained by GA, both designed for 100% load situation.

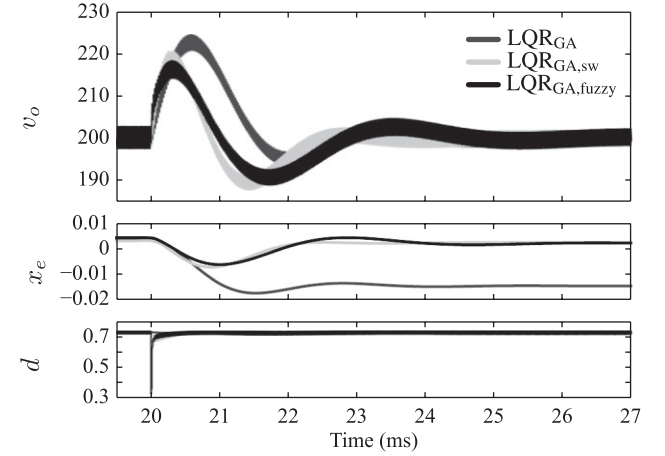


Fig. 10. Transient response to load disturbance from 100% to 75%.

The AND method uses the product of the membership values (μ) of the output current (i_o) and its derivative (i_o').

One advantage of using a fuzzy supervisor is that the signals φ_{100} , φ_{75} , φ_{50} and φ_{25} now can assume any value between 0 and 1, and the control signal is now obtained by a combination of each LQR controller action in (32), resulting in $\tilde{d}$ without sudden changes.

The performance of this strategy compared to others is shown in the next section.

VII. COMPARATIVE ANALYSIS

This section presents a detailed comparison between the controllers obtained from the solution of Problems 1 to 4.

First, in Fig. 9, one has a comparison between the single controllers from Problem 1 (designed without GA) and from Problem 2 (designed with GA). The load switches from 25% to 100% of the output power. The performance of the LQR obtained from Problem 2 is superior in terms of settling times, with the control design computational time given in Section VI.

In the comparisons in Figs 10 to 13 only the single controller from Problem 2 is used in order to have a fair

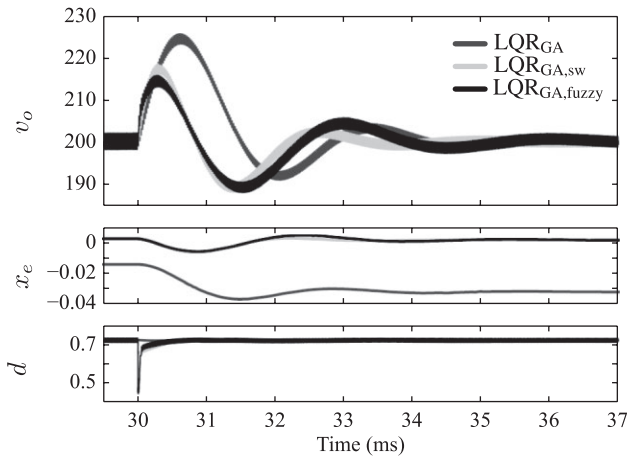


Fig. 11. Transient response to load disturbance from 75% to 50%.

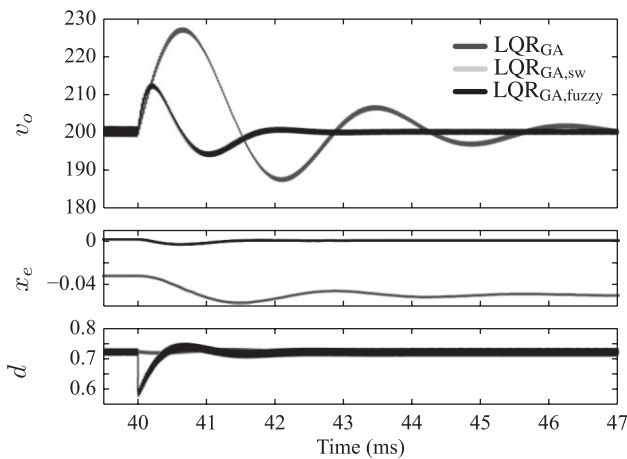


Fig. 12. Transient response to load disturbance from 50% to 25%.

comparison of performance of controllers designed with the help of the GA. Notice that these figures also show the integral of the tracking error (x_e) and the control signal (d), two critical variables for implementation.

The entire range of load is covered by the tests shown in Figs 10–13 for which one has load variations according to

$$\begin{cases} t < 20 \text{ ms} & P_o = 100\% \\ 20 \text{ ms} \leq t < 30 \text{ ms} & P_o = 75\% \\ 30 \text{ ms} \leq t < 40 \text{ ms} & P_o = 50\% \\ 40 \text{ ms} \leq t < 50 \text{ ms} & P_o = 25\% \\ t \geq 50 \text{ ms} & P_o = 100\% \end{cases} \quad (36)$$

In all cases, one can conclude that the output of the closed-loop system with the single controller from Problem 2 has the poorer performance, since it was designed for a single load condition (100% of output power). One can also see that

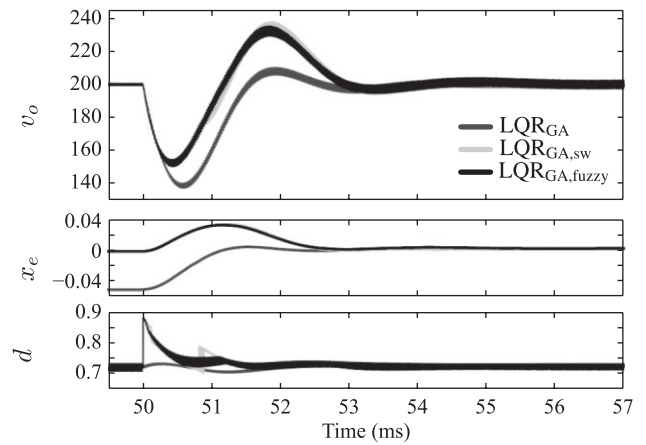


Fig. 13. Transient response to load disturbance from 25% to 100%.

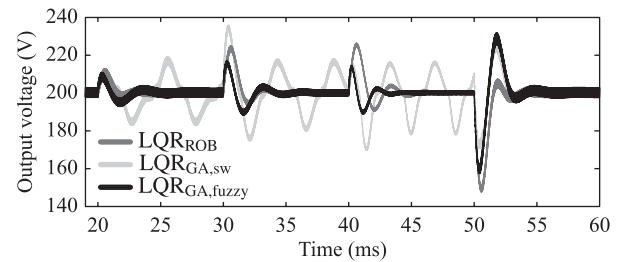


Fig. 14. Load perturbations to the threshold regions.

the switched controller from Problem 3 and the fuzzy logic based controller from Problem 4 have similar performance in these tests. However, a drawback of the switching control strategy becomes clear in Fig. 14, where one has load switch near the threshold values, as given by the load switching pattern

$$\begin{cases} t < 20 \text{ ms} & P_o = 100\% \\ 20 \text{ ms} \leq t < 30 \text{ ms} & P_o = 87.5\% \\ 30 \text{ ms} \leq t < 40 \text{ ms} & P_o = 62.5\% \\ 40 \text{ ms} \leq t < 50 \text{ ms} & P_o = 37.5\% \\ t \geq 50 \text{ ms} & P_o = 100\% \end{cases} \quad (37)$$

Notice that Fig. 14 also shows the results for this test with the robust controller from [18], using the weighting matrices entries from Table III, resulting in the gains

$$\mathbf{K}_{ROB} = [8.317 \times 10^{-3} \quad 525.23 \times 10^{-6} \quad -3.137]. \quad (38)$$

One can see that the switching controller has a dynamic response deteriorated when the load switches near the threshold values, and that the robust controller (38), based on a single gain, ensures good performance in all cases, as

expected, but has in general a poorer dynamic response when compared with the fuzzy logic based controller.

VIII. STABILITY ANALYSIS

The control strategies from Problems 1 to 4 demonstrate stable behavior for the entire load range in the tests shown in the previous section. However, a mathematical analysis of stability for system (16) in closed-loop for the controller provided by Problems 1 to 4 is important and will be addressed in this section.

First, note that the system (16) admits a polytopic representation (see [22]) given by

$$\dot{\xi} = \hat{F}(\alpha)\xi + \hat{G}(\alpha)\tilde{d} \quad (39)$$

where

$$\begin{aligned} (\hat{F}, \hat{G})(\alpha) &= \sum_{i=1}^4 \alpha_i (\hat{F}_i, \hat{G}_i) \\ \alpha_i &\geq 0, i=1, \dots, 4, \sum_{i=1}^4 \alpha_i = 1. \end{aligned} \quad (40)$$

Notice that the polytopic representation describes the dynamics of system (16) for any load value in Table IV, being the vertices given by the evaluation of matrices $(\hat{F}, \hat{G})_i$, $i=1, \dots, 4$ using (17) with the parameters in Table I for the values of load resistance in Table IV, corresponding to situations of 100%, 75%, 50%, and 25% of output power.

Consider now that the controller is given by another convex combination of the load control actions, that is,

$$\tilde{d} = \sum_{j=1}^4 \beta_j \tilde{d}_j, \beta_j \geq 0, j=1, \dots, 4, \sum_{j=1}^4 \beta_j = 1 \quad (41)$$

where $\tilde{d}_j, j=1, \dots, 4$ here represent the control actions $\tilde{d}_{100}$ (for the load situation of 100% of output power) to the control action $\tilde{d}_{25}$.

The control action in (41) can be seen as

$$\begin{aligned} \tilde{d} &= \sum_{j=1}^4 \beta_j K_j \xi = K(\beta) \xi \\ \beta_j &\geq 0, j=1, \dots, 4, \sum_{j=1}^4 \beta_j = 1. \end{aligned} \quad (42)$$

Thus, the closed-loop system can be rewritten as

$$\dot{\xi} = (\hat{F}(\alpha) - \hat{G}(\alpha)K(\beta))\xi \quad (43)$$

Notice all the situations of load in Table IV with all the situations of controllers in Problems 1 to 4 are covered by this representation.

The quadratic stability [22] in Theorem 1 certifies the stability of the closed-loop system (43) for arbitrary variations of load and for arbitrary variations of the controller covering the fixed controllers in Problems 1 and 2, and the switching controllers of Problem 3 and the fuzzy controller of Problem 4.

Theorem 1. System (43) is quadratically stable if there exists a matrix $P \in \mathbb{R}^{3 \times 3}$ such that

$$P = P^T > 0 \quad (44)$$

$$\begin{aligned} (\hat{F}_i - \hat{G}_i K_j)^T P + P(\hat{F}_i - \hat{G}_i K_j) &< 0 \\ i=1, \dots, 4, j=1, \dots, 4 \end{aligned} \quad (45)$$

Proof. If Theorem 1 has a solution, then

$$v(\xi) = \xi^T P \xi > 0 \quad \forall \xi \neq 0 \quad (46)$$

and, from the feasibility of (45) and due to convexity

$$\begin{aligned} \sum_{j=1}^4 \beta_j \sum_{i=1}^4 \alpha_i ((\hat{F}_i - \hat{G}_i K_j)^T P + P(\hat{F}_i - \hat{G}_i K_j)) \\ = (\hat{F}(\alpha) - \hat{G}(\alpha)K(\beta))^T P + P(\hat{F}(\alpha) - \hat{G}(\alpha)K(\beta)) < 0 \end{aligned} \quad (47)$$

which ensures that

$$\begin{aligned} \dot{v}(\xi) &= \xi^T (\hat{F}(\alpha) - \hat{G}(\alpha)K(\beta))^T P \\ &+ P(\hat{F}(\alpha) - \hat{G}(\alpha)K(\beta))\xi < 0 \quad \forall \xi \neq 0 \end{aligned} \quad (48)$$

for all α and β in unit simplex.

Thus, P in Theorem 1 is a matrix of a quadratic Lyapunov function ensuring the stability of the closed-loop system.

Notice that Theorem 1 is written with LMI conditions which are known to be solved in a fast and efficient way by interior point specialized algorithms as [28] and writing the stability analysis problem provides a suitable way to get a Lyapunov function for the system.

For the load values in Table IV, and for the controllers in Problems 2 to 4, one has that Theorem 1 provides

$$P = \begin{bmatrix} 13.2252 & 4.0302 & 0.0253385 \\ 4.0302 & 235.1117 & 0.0390536 \\ 0.0253385 & 0.0390536 & 0.0000945 \end{bmatrix} \times 10^{-3} \quad (49)$$

ensuring the stability of (43) in all cases of controllers and load changes.

This solution certifies the closed-loop stability only testing a finite number of LMIs given by Theorem 1, thus avoiding gridding procedures and local stability analysis by

means of exhaustive circuit simulation. Although the approach in Theorem 1 is not new in control theory, the use of such LMIs to certify the design of controllers for boost converters has not been addressed as reported here.

Finally, as discussed in [18], one has that the closed-loop system studied here is actually a bilinear system. Hence, the stability properties studied in this section are local. Once a controller for each linear model has been found, the states inside the region of attraction of the nonlinear system will satisfy a Lyapunov inequality as given in Section IV.B in [18].

IX. CONCLUSIONS

This paper presented a comparison between four different LQR strategies suitable for boost converters with large load range. As a first contribution, one has the use of a genetic algorithm to find local LQRs suitable to optimize the performance of a boost converter in terms of a given objective function for each load condition. The proposed GA evolves based on data obtained from an electrical circuit simulator, which takes into account nonlinearities and parasitic resistances. Another contribution for boost converter applications is the robust stability analysis of the closed-loop system carried out as in Section VIII. This design and validation procedure can help the control designer find automatically a set of suitable LQR controllers. The comparison of the controllers pointed out to the advantages of the fuzzy logic based strategy, even with respect to a robust LQR based on LMIs, being extensible to the control of other power converters.

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