

A Generalized Mathematical Analysis for Matrix Converters

Special Session on Matrix Converters

Alvaro Daniel Arioni

Department of Electrical Engineering - DEE
State University of Santa Catarina - UDESC
Joinville, Brazil
Email: alvaroarioni@gmail.com

Sérgio Vidal Garcia Oliveira, *Member, IEEE*

Department of Electrical Engineering - DEE
State University of Santa Catarina - UDESC
Joinville, Brazil
Blumenau Regional University - FURB, Brazil
Email: sergio_vidal@ieee.org

Abstract—Although being well known for more than thirty years, matrix converters still appear to be a cluster of special cases — either from topology or modulation point of view — targeted to achieve one stage frequency conversion without energy storage elements. This paper investigates the very basic principles behind the matrix converters, and concludes that they should be seen as total power converters rather than just frequency converters. It shows that traditional matrix converter applications can be completely modeled by a couple of equations first presented in 2001 regarding space vector modulation [1], which are ultimately equivalent to any configuration/modulation scheme, and whose only constraint is the energy conservation law itself. This analysis is based on the theory of functions of a complex variable, and is verified by simulation results.

I. INTRODUCTION

The matrix converter topology [2], [3], [4] has been object of thorough academic research mainly aiming one stage sinusoidal frequency conversion. That was the approach of classic works from Venturini [5], Huber [6] and others.

Not purely sinusoidal systems have been frequently treated as a matter of distorted or unbalanced supply voltage to cope with, and corresponding different compensation methods, either open [7] or closed [8], [9] loop to address them. Yet, due to the lack of major energy storage elements, this has been generally seen as a limitation [3] on matrix converter operation, which, as suggested ahead, may not be exactly so.

Applying the instantaneous values control theory [10] for feed forward compensation [7] of harmonic contaminated or asymmetrical input source voltage is relatively straightforward, though the effective realization of this control strategy [11] is not always simple.

Anyway, a close look to those works may suggest that the sinusoidal system is not actually the main core of matrix converters, but just an special case. Then, for the sake of generality, no sinusoidal hypothesis will be considered hereinafter.

This paper uses basic, though rigorous, concepts of complex algebra [12] to build a mathematical model which outlines the main properties of the matrix converter, not considering specific modulation or control. Even though some of these characteristics are implicit from other works [13], have been developed in deep related to the specific modulation strategy [14], [15], or have been settled up just for the sinusoidal

case, like [16], a similar simple and generalized mathematical modeling approach has not been presented so far.

It is the main goal of this paper to show the matrix converter as a flexible total power converter, instead of just a sinusoidal frequency converter, and to set out the mathematical framework for further generalizations, including vector optimization and reactive power transfer compensation and control.

It is also shown how the zero determinant transfer matrices are behind the classic space vector approach, so no vector diagram is needed to explain such modulation method at all.

Moreover, based on the mathematical model, it is proven that high frequency switched matrix converters are essentially equivalent to any other real or hypothetical instantaneous power converter, provided that at least some active power is delivered and over-modulation is avoided.

Finally, the conclusions from the mathematical analysis are verified through numerical simulation.

II. MODEL UNDER ANALYSIS

Since the main object of interest is the matrix of switches and the relationship among the input/output variables, the model is simply represented by voltage sources in one side and current sources in the other, as shown in Fig. 1.

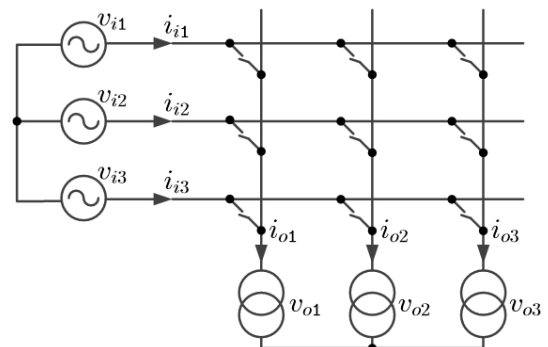


Figure 1. Matrix Converter

Thus, it resembles a variable speed motor drive, for example, being the inductive load approximately modeled by the current sources.

In order to avoid an open circuit for the current sources and a short circuit for the voltage sources, the matrix (Fig. 1) has

to contain exactly one conducting switch in each column all the time, no more, no less.

A. The Transfer Functions

The following matrix equations express the relationship among currents and voltages from the circuit in Fig. 1:

$$\begin{pmatrix} i_{i1} \\ i_{i2} \\ i_{i3} \end{pmatrix} = T \cdot \begin{pmatrix} i_{o1} \\ i_{o2} \\ i_{o3} \end{pmatrix} \quad \begin{pmatrix} v_{o1} \\ v_{o2} \\ v_{o3} \end{pmatrix} = T^T \cdot \begin{pmatrix} v_{i1} \\ v_{i2} \\ v_{i3} \end{pmatrix} \quad (1)$$

The 3×3 matrix T is virtually a hard copy of the physical matrix, as represented in figure 1, and its elements are either 0 for a blocking switch or 1 for a conducting switch.

Since each column of T must contain exactly one element equal to 1, there are just 27 different T matrices. Moreover, it should be noticed that the determinants of those 27 matrices are either 0, 1 or -1 . The 0 determinant matrices are of special interest.

III. VECTOR METHOD

The following linear transformation [17] is used to describe those transfer functions as space vectors in the complex plane:

$$\begin{pmatrix} x \\ x^* \\ x_0 \end{pmatrix} = S \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \text{ with :} \quad (2)$$

$$S = \frac{2}{3} \begin{pmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{pmatrix} \quad a = e^{\frac{2\pi i}{3}}$$

Applying transformation (2), equations (1) become:

$$\begin{pmatrix} i_i \\ (i_i)^* \\ i_{i0} \end{pmatrix} = S \cdot T \cdot S^{-1} \cdot \begin{pmatrix} i_o \\ (i_o)^* \\ i_{o0} \end{pmatrix} \quad (3)$$

$$\begin{pmatrix} v_o \\ (v_o)^* \\ v_{o0} \end{pmatrix} = S \cdot T^T \cdot S^{-1} \cdot \begin{pmatrix} v_i \\ (v_i)^* \\ v_{i0} \end{pmatrix}$$

A. Properties of The Complex Transfer Functions

- 1) Matrices $S \cdot T \cdot S^{-1}$ and $S \cdot T^T \cdot S^{-1}$ are transpose conjugates, which is valid for any real matrix T (not just transfer matrices) and for S as defined in (2).
- 2) For matrix T as defined in (1), the complex voltage transfer matrix $S \cdot T^T \cdot S^{-1}$ has a very special format:

$$A = S \cdot T^T \cdot S^{-1} = \begin{pmatrix} a_{11} & a_{12} & 0 \\ (a_{12})^* & (a_{11})^* & 0 \\ a_{31} & a_{32} & 1 \end{pmatrix} \quad (4)$$

- 3) These determinants are identical:

$$|T| = |T^T| = |S \cdot T \cdot S^{-1}| = |S \cdot T^T \cdot S^{-1}| \quad (5)$$

- 4) Based on the topology of the circuit (see Fig. 1), $i_{o0} = i_{o1} + i_{o2} + i_{o3} = 0$.

All these properties can be verified by straight algebra calculations.

B. The Complex Transfer Functions

Therefore, without any loss of generality, the complex transfer functions of the matrix converter are:

$$\begin{pmatrix} i_i \\ (i_i)^* \\ i_{i0} \end{pmatrix} = \begin{pmatrix} (a_{11})^* & a_{12} & (a_{31})^* \\ (a_{12})^* & a_{11} & (a_{32})^* \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} i_o \\ (i_o)^* \\ i_{o0} \end{pmatrix} \quad (6)$$

$$\begin{pmatrix} v_o \\ (v_o)^* \\ v_{o0} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & 0 \\ (a_{12})^* & (a_{11})^* & 0 \\ a_{31} & a_{32} & 1 \end{pmatrix} \cdot \begin{pmatrix} v_i \\ (v_i)^* \\ v_{i0} \end{pmatrix}$$

Because of the zero system decoupling and the conjugate redundancy, equations (6) are simplified to a couple of scalar complex equations:

$$i_i = (a_{11})^* \times i_o + a_{12} \times (i_o)^* \quad (7)$$

$$v_o = a_{11} \times v_i + a_{12} \times (v_i)^*$$

Equations (7) show the input-output relationships among whatever voltages and currents interacting with the converter. The parameters a_{11} and a_{12} depend exclusively on the switches matrix itself. A similar formalism has already been presented in [18].

IV. THE ZERO DETERMINANT CASE

For those matrices with $|T| = 0$, equations (7) can be further simplified. Following (5) and (4), the zero determinant means:

$$\begin{vmatrix} a_{11} & a_{12} \\ (a_{12})^* & (a_{11})^* \end{vmatrix} = 0 \Leftrightarrow |a_{11}| = |a_{12}| \quad (8)$$

In face of complex parameters of equal modulus, the polar form will be useful for re-writing equations (7) in this equivalent form:

$$i_i = \frac{\alpha}{2} (i_o e^{-i(\varphi+\psi)} + e^{i(\psi-\varphi)} (i_o)^*) \quad (9)$$

$$v_o = \frac{\alpha}{2} (v_i e^{i(\varphi+\psi)} + e^{i(\psi-\varphi)} (v_i)^*)$$

valid for some real parameters α , φ and ψ , dependant just on the switches matrix. Finally, equations (9) can be re-arranged as:

$$i_i = \frac{\alpha}{2} e^{-i\varphi} (e^{-i\psi} i_o + e^{i\psi} (i_o)^*) = \alpha e^{-i\varphi} \Re(e^{-i\psi} i_o)$$

$$v_o = \frac{\alpha}{2} e^{i\psi} (e^{i\varphi} v_i + e^{-i\varphi} (v_i)^*) = \alpha e^{i\psi} \Re(e^{i\varphi} v_i) \quad (10)$$

A. Properties of Functions when $|T| = 0$

- 1) Complex function $v_o = f(v_i)$ maps the entire complex plane to a straight line whose slope depends solely on the status of the switches matrix.
- 2) Complex function $i_i = g(i_o)$ maps the entire complex plane to another straight line whose slope also depends solely on the status of the switches matrix.
- 3) For those matrices with $|T| = 0$, α and angles φ and ψ can take the following values:

$$\varphi = \left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6} \right\} \quad (11)$$

$$\psi = \left\{ 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3} \right\} \quad \alpha = \left\{ 0, \frac{2}{\sqrt{3}} \right\}$$

which can be verified by simple calculations.

- 4) Among non zero ($\alpha \neq 0$) equations (10), those discrete values represents 36 different combinations of φ and ψ angles. However, since:

$$e^{i(\psi+\pi)} \Re \left(e^{i(\varphi+\pi)} v_i \right) = (-e^{i\psi}) \Re \left(-e^{i\varphi} v_i \right) = e^{i\psi} \Re \left(e^{i\varphi} v_i \right)$$

there are only 18 effectively different non zero combinations.

V. HIGH FREQUENCY COMMUTATION OF SWITCHES

The characteristics of section IV-A make the zero determinant matrices particularly useful. Matrices with $|T| = \pm 1$ might also be used, specially for improving low level commutation scheme [19], but they are not needed for the aim of the onward development.

Definition 1. Let $m_k \times w_r$, for $k = 0..5$, and $r = 0..5$ be the (real) slice of time when angle $\varphi_k = (2k+1)\frac{\pi}{6}$ is applied together with angle $\psi_r = \frac{\pi}{3}r$ and $\alpha = \frac{2}{\sqrt{3}}$ during the commutation cycle. Considering that $\alpha = 0$ will also be used part of the time during that cycle, then necessarily:

$$\sum_{k=0}^5 m_k \leq 1 \quad \text{and} \quad \sum_{r=0}^5 w_r \leq 1 \quad (12)$$

The input current and output voltage are obtained by adding up equations (10) per modulation cycle, in all possible angle configurations:

$$i_i(t) = \sum_{k=0}^5 \sum_{r=0}^5 m_k w_r \frac{2}{\sqrt{3}} e^{-i\frac{\pi(2k+1)}{6}} \Re \left(e^{-i\frac{\pi r}{3}} i_o \right) \quad (13)$$

$$v_o(t) = \sum_{r=0}^5 \sum_{k=0}^5 m_k w_r \frac{2}{\sqrt{3}} e^{i\frac{\pi r}{3}} \Re \left(e^{i\frac{\pi(2k+1)}{6}} v_i \right)$$

The summations (13) can be re-arranged as products of sums:

$$i_i(t) = \left(\sum_{k=0}^5 m_k \frac{2}{\sqrt{3}} e^{-i\frac{\pi(2k+1)}{6}} \right) \Re \left(\left(\sum_{r=0}^5 w_r e^{-i\frac{\pi r}{3}} \right) i_o \right) \quad (14)$$

$$v_o(t) = \left(\sum_{r=0}^5 w_r e^{i\frac{\pi r}{3}} \right) \Re \left(\left(\sum_{k=0}^5 m_k \frac{2}{\sqrt{3}} e^{i\frac{\pi(2k+1)}{6}} \right) v_i \right)$$

Definition 2. Let $M(t) = \sum_{k=0}^5 m_k \frac{2}{\sqrt{3}} e^{i\frac{\pi(2k+1)}{6}}$ and $W(t) = \sum_{r=0}^5 w_r e^{i\frac{\pi r}{3}}$, calculated for each time period.

Equations (14) can be re-written as these two simple expressions¹:

$$v_o(t) = W(t) \Re(M(t) v_i(t)) \quad (15)$$

$$i_i(t) = M(t)^* \Re(W(t)^* i_o(t))$$

which were first presented in [1] and completely describe the matrix converter behavior when using zero determinant matrices.

¹ $\Re(z)$ is the real part, and $\Im(z)$ the imaginary part of a complex value z

A. Remarks on equations (15)

- 1) The switched variables, $v_o(t)$ and $i_i(t)$, are expressed as functions of the continuous variables, $v_i(t)$ and $i_o(t)$.
- 2) Not surprisingly, $M(t)$ and $W(t)$ exchange places with the conjugate of each other between the two equations. This is a direct consequence of the transpose conjugate property of matrices in (3).
- 3) Since m_k and w_r are arbitrarily chosen time slices, then $M(t)$ and $W(t)$ are free of choice complex functions.
- 4) However, because of the physical restrictions (12), it must be $|M(t)| \leq 1$ and $|W(t)| \leq \frac{\sqrt{3}}{2}$, or briefly:

$$|M(t)| \cdot |W(t)| \leq \frac{\sqrt{3}}{2} \quad (16)$$

which in fact constitutes the only real drawback of the matrix converter as a full purpose power converter.

- 5) Dismissing the "input" and "output" subscripts, the equations can model a current-source matrix converter feeding a capacitive load as well. Voltages and currents are mutually symmetric.
- 6) Equations (15) are valid no matter what kind of functions the input-output currents and voltages are. They need not to be sinusoidal, nor even periodic.
- 7) Neither the source, nor the load, have to be "balanced". Even three to single phase systems, like [20], can be seen as an special case with ∞ impedance in one load branch.

VI. THE TOTAL CONVERTER

Equations (15) suggest that matrix converters may have higher intrinsic flexibility than just that of frequency conversion. Thus, distorted voltages or currents could, for example, be absorbed or generated as needed. That matter will be deeply analyzed hereinafter.

Theorem 3. Any power converter system, matrix or not, without energy storage, can be modeled by equations (15) whenever there is at least some active power transfer.

Proof: (outline) Let $v_i(t)$, $v_o(t)$, $i_i(t)$ and $i_o(t)$ be voltage and current complex vectors generated by some power conversion system. Functions $M(t) = m_1 + i \cdot m_2$ and $W(t) = w_1 + i \cdot w_2$ must be found such that equations (15) are satisfied. Thus, the four unknowns m_1 , m_2 , w_1 and w_2 have to be uncovered.

Splitting in real and imaginary parts, the two equations (15) turn into the following fourth order real equations system:

$$\begin{bmatrix} \Re(v_i) & -\Im(v_i) & 0 & 0 \\ 0 & 0 & \Re(v_i) & -\Im(v_i) \\ \Re(i_o) & 0 & \Im(i_o) & 0 \\ 0 & -\Re(i_o) & 0 & -\Im(i_o) \end{bmatrix} \cdot \begin{bmatrix} m_1 w_1 \\ m_2 w_1 \\ m_1 w_2 \\ m_2 w_2 \end{bmatrix} = \begin{bmatrix} \Re(v_o) \\ \Im(v_o) \\ \Re(i_i) \\ \Im(i_i) \end{bmatrix}$$

The system is solved in two stages. On the first stage it is analyzed as if the unknowns were $m_1 w_1$, $m_2 w_1$, $m_1 w_2$ and $m_2 w_2$. Thus it becomes a linear system, of rank 3, with one degree of freedom and infinite solutions under the condition:

$$\Im(i_i) \Im(v_i) + \Re(i_i) \Re(v_i) = \Im(i_o) \Im(v_o) + \Re(i_o) \Re(v_o)$$

This condition is nothing less than the energy conservation law $\Re(v_i(i_i)^*) = \Re(v_o(i_o)^*)$, which is true because of the hypothesis of no energy storage.

On the second stage, the values of m_1, m_2, w_1 and w_2 must be taken from the products m_1w_1, m_2w_1, m_1w_2 and m_2w_2 found on the first stage. Logarithms can be used to turn it again into a real linear system:

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \log(m_1) \\ \log(m_2) \\ \log(w_1) \\ \log(w_2) \end{pmatrix} = \begin{pmatrix} b \\ c \\ d \\ e \end{pmatrix}$$

Once more, this new linear system has rank 3. Substituting the results from the first stage, the values of m_1, m_2, w_1 and w_2 may always be uncovered under the condition:

$$\Im(i_o) \Im(v_o) + \Re(i_o) \Re(v_o) = \Re(v_o(i_o)^*) \neq 0$$

And this completes the main (outline) of the proof. ■

This fact is quite remarkable because it shows that the matrix converter basic equations (15) can replicate any other real or hypothetical instantaneous power converter, provided that there is at least some active power transfer. The corresponding matrix converter will be physically feasible just if condition (16) manages to be true.

VII. ACTIVE AND REACTIVE POWER

The active power delivered through the matrix converter can be calculated following the equations (15) pattern. From either $\frac{3}{2} \cdot \Re(v_i(t) \cdot i_i(t)^*)$ or $\frac{3}{2} \cdot \Re(v_o(t) \cdot i_o(t)^*)$ active power formulas, the next expression is deduced:

$$P = 1.5 \cdot \Re(M(t)v_i(t)) \times \Re(W(t)^*i_o(t)) \quad (17)$$

The close relationship between active power equation (17) and general matrix equations (15) is clear.

A. Reactive Power in Each Side

Different from active power, the reactive power need not to be equal at both sides of the matrix converter, since there is not such thing as "reactive energy conservation".

Definition 4. Let $Q_i = \frac{3}{2} \cdot \Im(v_i(t) \cdot i_i(t)^*)$ be the reactive power at the input side, and $Q_o = \frac{3}{2} \cdot \Im(v_o(t) \cdot i_o(t)^*)$ be the reactive power at the output side.

The following expressions are deduced:

$$\begin{aligned} Q_i &= 1.5 \cdot \Im(M(t)v_i(t)) \times \Re(W(t)^*i_o(t)) \\ Q_o &= -1.5 \cdot \Re(M(t)v_i(t)) \times \Im(W(t)^*i_o(t)) \end{aligned} \quad (18)$$

Again, there is a clear pattern relationship among equations (18), (17) and (15). Altogether, those equations manage to describe the main properties of the matrix converter, which is useful to achieve a better understanding of it.

For example, using the traditional SVM (Space Vector Modulation), there must always be some active power transfer for reactive power to be delivered [21]. However, $M(t)$ and $W(t)$

need not to be traditional "continuous" functions, and thus they could be built to get non zero reactive power even without active power delivery, still complying with the implications of the mathematical model. Indeed, [22] suggested a different modulation schema for pure reactive power delivery through the converter. Therefore, the reactive power problem happens to be a limitation of the modulation schema (SVM) rather than a drawback of the matrix converter itself.

VIII. MODULATION AND CONTROL STRATEGIES

The first step to achieve an adequate control system is developing a good model. General equations (15), (17) and (18) can be used as mathematical basis for any control or modulation approach [23], and thus help to reduce the sense of complexity of matrix converters.

A. Two Stages Vector Modulation

This is the most straightforward application of the already analyzed mathematical model. Suppose a voltage-source system. First, in the rectifying stage, $M(t)$ is chosen to make $\Re(M(t)v_i(t))$ equal to some constant value K . Because of condition (16) it should be $K \leq \min(|v_i(t)|) \forall t$. Then, in the inverting stage, $W(t)$ is used to build $v_o(t)$ as desired.

As stated in section V-A, a current-source system could be treated equally.

This modulation strategy relies on instantaneous values control theory, as mentioned in section I, and it should be clear that any $v_o(t)$ output format could thus be obtained from any $v_i(t)$ input format, just regarding (16) amplitude restriction.

Case 1. As a particular instance, a system with distorted and unbalanced complex input voltage $v_i(t) = v(t)e^{i \cdot w_i(t)}$ will be analyzed to produce an output sinusoidal system $v_o(t) = V_o e^{i \cdot w_o \cdot t}$, over a linear load, thus being the output current $i_o(t) = I_o e^{i(w_o \cdot t - \delta_{\text{cst}})}$. Moreover, a displacement factor θ_{cst} is desired for the input current.

Since $M(t)$ and $W(t)$ are free of choice, they are selected as:

$$M(t) = \frac{V_i e^{i(\theta_{\text{cst}} - w_i(t))}}{v(t)} \quad W(t) = \frac{\sqrt{3}}{2} e^{i \cdot w_o \cdot t} \quad (19)$$

which comply with condition (16) when $V_i \leq \min(v(t)) \forall t$. Thus $M(t)$ and $W(t)$ can be PWM built as described in section V, definition 2.

Then, equations (15) are applied to get:

$$\begin{aligned} v_o(t) &= \frac{\sqrt{3}}{2} V_i \cdot \cos(\theta_{\text{cst}}) e^{i \cdot w_o \cdot t} \\ i_i(t) &= \frac{\sqrt{3} \cdot V_i \cdot I_o \cdot \cos(\delta_{\text{cst}}) e^{i(w_i(t) - \theta_{\text{cst}})}}{2 \cdot v(t)} \end{aligned} \quad (20)$$

Not only a completely distorted input voltage has turn into a sinusoidal output voltage, but an arbitrary displacement input factor θ_{cst} (inductive or capacitive) has been enforced as well.

A detailed simulation model, using this modulation strategy, has been built employing a Modelica/SystemModeler® environment in order to verify how this direct feed forward compensation effectively works. The results are shown in section IX.

Equation (17) is used to find the active power delivered through the converter:

$$P = \frac{3\sqrt{3}}{4} V_i \cdot I_o \cos(\delta_{\text{cst}}) \cos(\theta_{\text{cst}}) \quad (21)$$

Likewise, equations (18) evaluate the reactive power at the input and the output sides:

$$Q_i = \frac{3\sqrt{3}}{4} V_i \cdot I_o \cos(\delta_{\text{cst}}) \sin(\theta_{\text{cst}}) \quad (22)$$

$$Q_o = \frac{3\sqrt{3}}{4} V_i \cdot I_o \sin(\delta_{\text{cst}}) \cos(\theta_{\text{cst}})$$

B. Remarks on Power Transfer

- 1) Equations (22) show that an inductive load can be easily converted to a capacity input current.
- 2) The closer to zero the δ_{cst} and θ_{cst} angles are, the larger the amount of power delivered (see equation (21)).
- 3) On the other hand, the matrix converter with SVM modulation does not behave too well when angles δ_{cst} and θ_{cst} are near $\pm\frac{\pi}{2}$, since both active and reactive power tend to zero.
- 4) The main drawback of this SVM modulation for large angles is related to equation (20), where θ_{cst} near $\pm\frac{\pi}{2}$ leads to $v_o(t)$ close to zero.
- 5) Whereas a large δ_{cst} depicts a physical constraint (perhaps a motor at no load), $v_o(t)$ close to zero due to large θ_{cst} is a modulation constraint that could be overcome through either control [24], pure modulation [22], or even predictive control [25].

IX. SIMULATION RESULTS

The main purpose of the simulation work was to check the mathematical model and to verify the total conversion capacity of the converter. The simulation model, mounted on a Modelica/SystemModeler[®] environment, admits any three-phase input source, and generates a three-phase output, following any other three-phase reference pattern.

The built model took into account the on-off conductance of the switches, the switching losses, the switching delays and followed an strict safe commutation strategy [26], in order to endorse the feasibility of running the matrix as an all-purpose converter. However, the details of the Modelica model implementation are beyond the scope of this paper.

Moreover, the triangular and cubic sinusoidal sources used in simulations were chosen to illustrate the theoretical point, even though not connected with any practical problem.

In order to get close to the most common applications, the simulation was prepared with a linear balanced load. However, the model equally runs with unbalanced or even "virtual single phase" loads (see point 7, section V-A), thus resembling [20].

A. Simulation Data

- Three-phase, 50 Hz, input source with mixed waveforms in each phase: triangular, sinusoidal and cubic sinusoidal. The three input phases contain even DC components.
- Balanced linear RL load, with 4 Ω and 7,7 mH.
- Converter switching frequency: 5 kHz.

- Three-phase balanced output voltages, with 200 V peak value and frequency of 77 Hz.
- Input displacement factor 0° to achieve better power delivery capacity, as pointed out in section VIII-B.

B. Input Side

The simulated voltages and currents from the input side are shown in Fig. 2.

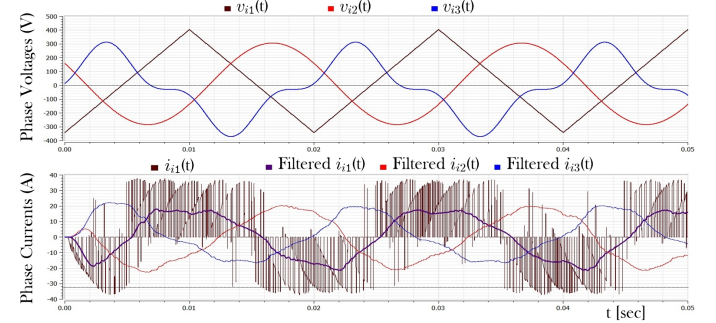


Figure 2. Simulated input voltages of the three phases, input switched current of phase 1 and filtered input currents of the three phases

Just phase 1 switched current is shown in Fig. 2 in order to make the graphics readability easier. Following equation (20), the (filtered) input currents were not expected to be sinusoidal, but they clearly comply with the zero displacement factor stated in the requirements.

Given a sinusoidal output (voltage/current) system, and non sinusoidal input voltage, theory shows through equations (15) that the input current cannot be improved. Then, it can be said that the matrix converter gives harmonics back to the source.

C. Output Side

The simulated voltages and currents from the output side are shown in Fig. 3.

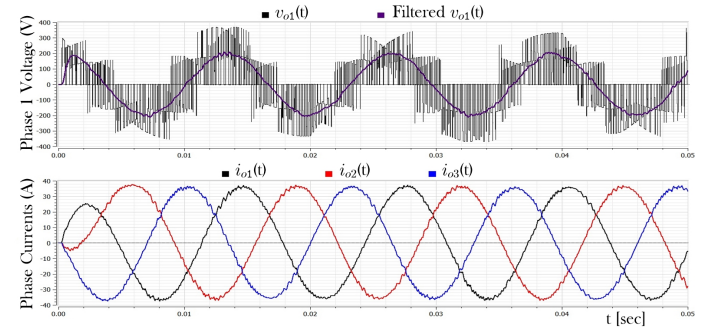


Figure 3. Output voltage of phase 1 and the three-phase output currents

The sinusoidal output goal is achieved even with a not so high switching frequency (5 kHz). The displacement output factor corresponds to the physical characteristics of the load. The same results are obtained with any other input waveform, provided that relationship (16) is fulfilled.

Fig. 4 illustrates the angles $\varphi = \arg(M(t))$ and $\psi = \arg(W(t))$ along with $|M(t)|$ to show how this feed forward compensation works for any input waveform. To achieve sinusoidal output $|W(t)|$ must be constant, so it is of no interest to be shown.

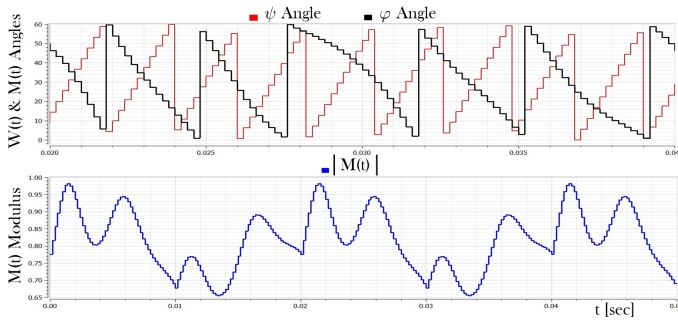


Figure 4. Angles $\psi = \arg(W(t))$, $\varphi = \arg(M(t))$ and modulus $|M(t)|$

As long as $|M(t)|$ remains lower than one, there will be no over-modulation and the sinusoidal output will be straightforward.

X. CONCLUSIONS

This paper presented a complete mathematical framework for the matrix converter based on the compact complex functions paradigm. It showed how the zero determinant condition is behind the traditional vector approach, and further developed equations presented in [1].

Moreover, it was proven that matrix converters can replicate any other real or hypothetical instantaneous power converter, provided that there is at least some active power transfer. This means that distorted voltages or currents could be absorbed or generated as needed, enabling its use as possible filter, dynamic voltage restorer or impedance adapter, for instance, rather than just the traditional frequency conversion operation.

Finally, the accuracy of the mathematical analysis was verified through numerical simulations, where whichever input source (triangular for instance) managed to turn into balanced sinusoidal output, as predicted by the equation model.

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