

Expanded Space Vector Modulation of Direct Matrix Converters Including Hidden Rotating Pairs

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Abstract—A novel version of the traditional SVM technique, with the integration of the rotating vectors, hereinafter referred to as Expanded Space Vector Modulation (ESVM), is introduced in this paper. Although different ways to benefit from the rotating vectors' configuration have already been proposed in the literature, the generalization of the SVM theory to include them has not been proposed so far. The rotating vectors are grouped in rotating pairs, which are demonstrated to behave, by all practical aspects, as any other stationary vector. As a consequence, a circular modulation style is developed, which comprises increased flexibility over the traditional approach, thus improving the Direct Matrix Converter (DMC) capabilities. The new technique is used to achieve safe commutation, even though it might have been also applied to improve the switching losses as well. Besides the theoretical analysis, simulation and experimental results are shown to support the feasibility of the approach.

Index Terms—AC-AC power conversion, Circular modulation, ESVM, Matrix converters, Rotating pairs, Safe commutation

I. INTRODUCTION

INHERITED from the Voltage Source Inverter (VSI), the Space Vector Modulation (SVM) has been widely used through a variety of power electronics applications. Starting from Huber work [1], the space vector representation has been a recurring reference point for matrix converter (MC) research too. Although many other modulation techniques have been proposed — with emphasis on the duty cycle methods, initially introduced by Venturini and Alesina [2] in the early 80's — more flexibility and a higher level of abstraction have granted SVM a prominent position among modulation techniques.

When originally introduced for VSIs, the geometric representation of the different configuration of switches was very intuitive and included just zero vectors and stationary vectors, which in turn ensured a repetitive pattern of commutations produced by the technique. The scope widening of vector representation methods to include matrix converters, as proposed in [1], was therefore a somewhat natural step forward.

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Like in the VSI case, most space vector approaches use just zero and stationary vectors too, as presented in [1], [3], [4] and many others. Yet, due to its more generic and flexible topology, a special subset of the MCs — the Direct Matrix Converter (DMC), Fig. 1 — includes a new type of space vectors which spin synchronized with the source, the so called "rotating vectors".

When assuming that rotating vectors are not used, the switch count of matrix converters can be reduced, allowing for different types of Indirect Matrix Converter (IMC) topologies [4], [5], [6], which can achieve an equally effective SVM.

On the other hand, some authors have sought for certain kind of usefulnesses for the rotating vectors which could exploit its special characteristics. For example, In order to minimize switching losses, [7] applies the physics principle of limiting the switching voltages among steps. Thus, the switching sequence within the modulation period is re-ordered to increase, step by step, each arm voltage. The final sequence is not a classical SVM sequence anymore, where the switching states corresponding to the rotating vectors naturally arise.

Also aiming for better switching losses, [8] departs from the traditional SVM calculations and then, depending on the measured input voltages, the output-phase switching states are combined, within the modulation period, to form a pulse sequence where a sequential commutation is always realized to the nearest input voltage, so minimizing switching losses. Some of the re-sorted switching states come out to be SVM rotating vectors. In order to improve voltage harmonics, as well as switching losses, [9] proposes to employ the traditional stationary, along with the nearest rotating vector in each sector.

Zero common-mode voltage (CMV), in case of balanced input, is a special property of the rotating vectors' switching state, which led several works to propose the reduction, or even elimination, of the matrix converter CMV. Based on the fact that the sum of the three clockwise or counterclockwise vectors is always zero, [10] and [11] propose to apply those triples instead of the zero vectors, reducing the CMV, but increasing the number of commutations. Based on the Singular Value Decomposition (SVD) strategy [12], a CMV reduction is achieved through a mathematical transformation process which includes the rotating vectors [13], whereas a perfect zero CMV is proposed using exclusively rotating vectors in [14].

Open-end-winding (OEW) drives, modulated with rotating vectors, are particularly attractive because they theoretically manage to fully remove the CMV [15]. Instantaneous CMV at

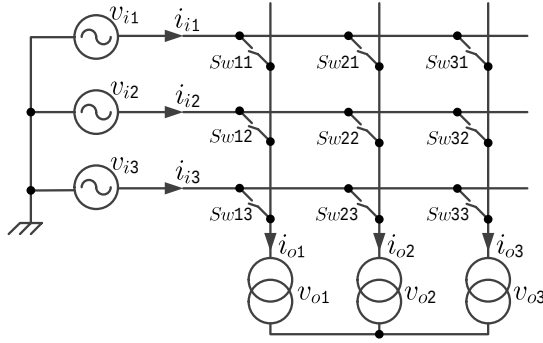


Fig. 1. Direct Matrix Converter.

OEW AC Machine terminals are eliminated with dual-matrix topologies, using exclusively rotating vectors in [16], [17], yet achieving 1.5 voltage gain, in contrast to 0.5 gain in [14].

The model predictive control (MPC) strategy [18] has also been proposed for matrix converter drives [19], [20], [21]. When applied to DMCs, this technique inherently uses every switching state, including those states corresponding to the rotating vectors. A combination of MPC and SVM, as proposed in [22], leads to switching sequences which include the rotating vectors, while starting and ending at zero vectors. Zero CMV is achieved in [23] through a finite control set MPC method, which only includes the six rotating vectors.

Anyhow, most of the modulation and control approaches for DMCs do not usually apply the rotating vectors' switching states, which were reputed to be rather useless for that purpose so far [24], [25], [26]. This paper inserts the rotating vectors, as "rotating pairs", in the space vector modulation theory on its own right, in a simple and elegant way. The result is a natural expansion of the SVM technique with straight improvements in flexibility, safe commutation and possibly in switching losses and CMV, among others, as well.

As a consequence of this SVM expansion, hereinafter named ESVM, a novel "circular modulation" method is presented along with some of its many possible usages. Finally, the novel technique is applied to improve the robustness of the matrix bidirectional-switches commutation, which is verified through analytic examination, numerical simulations and practical experimentation on a laboratory prototype.

II. THEORETICAL FOUNDATIONS

A DMC can be schematically depicted by Fig. 1 circuit. Thus, it can be thought of, for instance, as a variable speed drive, where the inductive loads are roughly modeled by the current sources [1]. There must always be exactly one conducting switch in each column in order to avoid shorting the voltage sources or opening the current sources.

As introduced in [3] the voltage and current matrix equations representing the circuit in Fig. 1 are:

$$\begin{pmatrix} i_{i1} \\ i_{i2} \\ i_{i3} \end{pmatrix} = T \cdot \begin{pmatrix} i_{o1} \\ i_{o2} \\ i_{o3} \end{pmatrix} \quad \begin{pmatrix} v_{o1} \\ v_{o2} \\ v_{o3} \end{pmatrix} = T^T \cdot \begin{pmatrix} v_{i1} \\ v_{i2} \\ v_{i3} \end{pmatrix} \quad (1)$$

The 3×3 matrices named " T " are virtually a hard copy of the physical switches matrix, as depicted in Fig. 1, where the elements are either 1 for a conducting switch or 0 for a

blocking switch. Since each " T " column must contain exactly one element equal to 1, there are just 27 different " T " matrices.

It is interesting to notice that the determinants of those " T " matrices can be either 0 for stationary and 1 or -1 for clockwise or counterclockwise rotating vectors' configurations.

A. Average Duty Cycle Transfer Functions

Employing the Pulse Width Modulation (PWM) technique, the high frequency components can be neglected and the 0's and 1's from " T " matrices in (1) revert to average d_{ij} values representing the Sw_{ij} switches (from Fig. 1) duty cycles, yielding new " D " average matrices (2). In order to avoid shorting the inputs or opening the outputs [1], now it is the sum of the three " D " matrices columns which must equal 1.

$$D = \begin{pmatrix} d_{11} & d_{21} & d_{31} \\ d_{12} & d_{22} & d_{32} \\ d_{13} & d_{23} & d_{33} \end{pmatrix} \quad \sum_{j=1}^3 d_{ij} = 1 \quad \forall i, j \quad 0 \leq d_{ij} \leq 1 \quad (2)$$

As a result, the general DMC PWM transfer functions are:

$$\begin{pmatrix} i_{i1} \\ i_{i2} \\ i_{i3} \end{pmatrix} = D \cdot \begin{pmatrix} i_{o1} \\ i_{o2} \\ i_{o3} \end{pmatrix} \quad \begin{pmatrix} v_{o1} \\ v_{o2} \\ v_{o3} \end{pmatrix} = D^T \cdot \begin{pmatrix} v_{i1} \\ v_{i2} \\ v_{i3} \end{pmatrix} \quad (3)$$

The complex linear transformation (4) is then applied to equations (3) in order to obtain the transfer functions in terms of the voltage input/output phasors v_i , v_o , and the current input/output phasors i_i , i_o , in the complex plane:

$$\begin{pmatrix} x \\ x^* \\ x_0 \end{pmatrix} = S \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad , \text{with } S \text{ defined as:} \quad (4)$$

$$S = \frac{2}{3} \begin{pmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{pmatrix} \quad a = e^{\frac{2\pi i}{3}}$$

Thus, the space vector representation of the DMC is derived:

$$\begin{pmatrix} i_i \\ (i_i)^* \\ i_{i0} \end{pmatrix} = S \cdot D \cdot S^{-1} \cdot \begin{pmatrix} i_o \\ (i_o)^* \\ i_{o0} \end{pmatrix} \quad (5)$$

$$\begin{pmatrix} v_o \\ (v_o)^* \\ v_{o0} \end{pmatrix} = S \cdot D^T \cdot S^{-1} \cdot \begin{pmatrix} v_i \\ (v_i)^* \\ v_{i0} \end{pmatrix}$$

Matrices $S \cdot D \cdot S^{-1}$ and $S \cdot D^T \cdot S^{-1}$ are transpose conjugates, which is valid for any real matrix D for S as defined in (4). Moreover, $S \cdot D^T \cdot S^{-1}$ matrix has always a very special format:

$$S \cdot D^T \cdot S^{-1} = \begin{pmatrix} a_{11} & a_{12} & 0 \\ (a_{12})^* & (a_{11})^* & 0 \\ a_{31} & a_{32} & 1 \end{pmatrix} \quad (6)$$

After simplifying (5) by means of the conjugate redundancy and the zero system decoupling — v_{i0} multiplied by 0 because of (6) third column, and $i_{o0} = i_{o1} + i_{o2} + i_{o3} = 0$ because of Fig. 1 topology — two scalar complex equations are deduced:

$$v_o = a_{11} \times v_i + a_{12} \times (v_i)^* \quad (7)$$

$$i_i = (a_{11})^* \times i_o + a_{12} \times (i_o)^*$$

Through high frequency commutation of switches, the complex coefficients a_{11} and a_{12} can be freely changed, and equations (7), involving the v_i , v_o , i_i and i_o complex phasors, in fact represent the full matrix converter capability.

Those equations, as a general model of the input-output relationships in a three phase matrix converter, were originally presented by Casadei in [27], even though that work gave credit to [3] which actually used a similar formalism to derive just the zero determinant case [28] of equations (7).

B. The Zero Determinant Case

Even though the " D " matrices, as defined in section II-A, are physically built from " T " matrices, the set of " D " matrices is mathematically more general. While there exist just 27 different " T " matrices, the number of possible " D " matrices is infinite. In fact, by the formal definition, every " T " matrix is always also a " D " matrix, and the theory will be developed over " D " matrices hereinafter, as if they were representing *virtual* instant switch values, as well as the " T " matrices represent *real* instant switch values. The following identities can be verified by straight algebra calculations:

$$|D| = |S.D.S^{-1}| = |S.D^T.S^{-1}| = |a_{11}|^2 - |a_{12}|^2 \quad (8)$$

When $|D| = 0$, then $|a_{11}| = |a_{12}|$ and, with the aid of polar form, (7) can be further simplified [3] to the next equations:

$$v_o = W \times \text{Re}(M \times v_i) \quad (9)$$

$$i_i = M^* \times \text{Re}(W^* \times i_o)$$

As well as a_{11} and a_{12} , the complex values M and W depend exclusively on the duty cycle matrix " D ", and not on the phasors v_i , v_o , i_i and i_o , which means that equations (9) map the entire complex plane to straight lines whose slopes depend solely on the duty cycle components of that matrix.

As aforementioned in section II, the 21 traditional zero and stationary SVM vectors are represented by zero determinant " T " matrices, thereby following the (9) equation format. All the same, any other zero determinant " D " matrix could be considered as representing a virtual stationary vector, obtained by a modulation method, hereinafter called "2nd order SVM".

The set of traditional stationary vectors are most useful for the SVM technique because they naturally entail a repetitive pattern of commutations. Some of that virtual stationary vectors, obtained by a 2nd order SVM, are special and can join the set to advantageously constitute an expanded SVM technique.

C. The Rotating Pair Vectors

Rotating vectors determinants are either 1 or -1 . However, half the sum of a clockwise and a counterclockwise rotating vectors always results in a determinant equal to zero, as it can be easily verified. Using the matrix format, for example:

$$\frac{1}{2} \left(\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \end{pmatrix} \quad (10)$$

This should be quite obvious also because both vectors are rotating at the same speed in opposite directions. In any case, resulting from the 2nd order SVM, the vector represented in

Table I
ROTATING PAIRS VOLTAGE TRANSFER FUNCTIONS

$\{1, 3, 2\}\{1, 2, 3\}$	$e^{i0} \times \text{Re}(e^{i0} \times v_i)$
$\{3, 2, 1\}\{3, 1, 2\}$	$e^{i0} \times \text{Re}(e^{i\frac{2\pi}{3}} \times v_i)$
$\{2, 3, 1\}\{2, 1, 3\}$	$e^{i0} \times \text{Re}(e^{i\frac{4\pi}{3}} \times v_i)$
$\{1, 3, 2\}\{3, 1, 2\}$	$e^{i\frac{\pi}{3}} \times \text{Re}(e^{i\frac{\pi}{3}} \times v_i)$
$\{3, 2, 1\}\{2, 3, 1\}$	$e^{i\frac{\pi}{3}} \times \text{Re}(e^{i\pi} \times v_i)$
$\{1, 2, 3\}\{2, 1, 3\}$	$e^{i\frac{\pi}{3}} \times \text{Re}(e^{i\frac{5\pi}{3}} \times v_i)$
$\{3, 1, 2\}\{2, 1, 3\}$	$e^{i\frac{2\pi}{3}} \times \text{Re}(e^{i0} \times v_i)$
$\{1, 3, 2\}\{2, 3, 1\}$	$e^{i\frac{2\pi}{3}} \times \text{Re}(e^{i\frac{2\pi}{3}} \times v_i)$
$\{1, 2, 3\}\{3, 2, 1\}$	$e^{i\frac{2\pi}{3}} \times \text{Re}(e^{i\frac{4\pi}{3}} \times v_i)$

equation (10) is an example of a virtual stationary vector, as defined in section II-B, just as any other vector built likewise.

Since the DMC comprises three clockwise and three counterclockwise rotating vectors, following the aforementioned pattern there are exactly nine virtual stationary "rotating pairs", defined as half the sum of a clockwise rotating vector and a counterclockwise rotating vector.

To have an easier description of those vectors, each matrix switching state will be named by a triple index, indicating the position of the conducting switch in each column. Consequently, the rotating pairs will be identified by a couple of triples. Following (9) equation format, they synthesize the nine constant slope complex transfer functions shown in Table I.

The so defined rotating pairs are therefore stationary vectors, mathematically analogous to the 18 traditional SVM stationary vectors [29]. On behalf of clarity, the traditional SVM vectors will also be indicated by an equivalent triple index hereinafter.

D. The Whole Picture: Joining the SVM Stationary Set

As presented in [28] and [3] works, the discrete set of equations, associated with the traditional SVM stationary vectors, can also be mathematically expressed in (9) format:

$$v_o = \frac{2}{\sqrt{3}} e^{i\psi_k} \times \text{Re}(e^{i\gamma_n} \times v_i) \quad , \text{for } \gamma_n \text{ and } \psi_k: \quad (11)$$

$$\gamma_n = \left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6} \right\}$$

$$\psi_k = \left\{ 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$$

Angle parameters γ_n and ψ_k in (11) equations yield $6 \times 6 = 36$ different combinations. However, since for any γ and ψ :

$$e^{i(\psi+\pi)} \text{Re}(e^{i(\gamma+\pi)} v_i) = -e^{i\psi} \text{Re}(-e^{i\gamma} v_i) = e^{i\psi} \text{Re}(e^{i\gamma} v_i) \quad (12)$$

there are just 18 effectively different stationary vectors. Likewise, the 9 rotating pairs, also correspond to 18 discrete combination of γ_n and ψ_k angle parameters. Following (9) pattern, the current equations are analogous. It is also interesting to notice that, while sharing the same ψ_k discrete angles, there is a $\frac{\pi}{6}$ phase-shift between the γ_n angles of these stationary vectors, and those of the rotating pairs as defined in Table I.

Table II
COMPLETE SET OF DMC SWITCHING STATES ACCORDING TO (INPUT) γ_n AND (OUTPUT) ψ_k ANGLE PARAMETERS

$\gamma_n \backslash \psi_k$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$
0	{1, 3, 2} {1, 2, 3}	{1, 2, 2}	{2, 2, 2}	{3, 2, 2}	{3, 2, 1} {3, 1, 2}	{3, 1, 1}	{1, 1, 1}	{2, 1, 1}	{2, 3, 1} {2, 1, 3}	{2, 3, 3}	{3, 3, 3}	{1, 3, 3}
$\frac{\pi}{3}$	{1, 1, 1}	{1, 1, 2}	{1, 3, 2} {3, 1, 2}	{3, 3, 2}	{3, 3, 3}	{3, 3, 1}	{3, 2, 1} {2, 3, 1}	{2, 2, 1}	{2, 2, 2}	{2, 2, 3}	{1, 2, 3} {2, 1, 3}	{1, 1, 3}
$\frac{2\pi}{3}$	{3, 1, 2} {2, 1, 3}	{2, 1, 2}	{2, 2, 2}	{2, 3, 2}	{1, 3, 2} {2, 3, 1}	{1, 3, 1}	{1, 1, 1}	{1, 2, 1}	{1, 2, 3} {3, 2, 1}	{3, 2, 3}	{3, 3, 3}	{3, 1, 3}
π	{1, 1, 1}	{2, 1, 1}	{2, 3, 1} {2, 1, 3}	{2, 3, 3}	{3, 3, 3}	{1, 3, 3}	{1, 3, 2} {1, 2, 3}	{1, 2, 2}	{2, 2, 2}	{3, 2, 2}	{3, 2, 1} {3, 1, 2}	{3, 1, 1}
$\frac{4\pi}{3}$	{3, 2, 1} {2, 3, 1}	{2, 2, 1}	{2, 2, 2}	{2, 2, 3}	{1, 2, 3} {2, 1, 3}	{1, 1, 3}	{1, 1, 1}	{1, 1, 2}	{1, 3, 2} {3, 1, 2}	{3, 3, 2}	{3, 3, 3}	{3, 3, 1}
$\frac{5\pi}{3}$	{1, 1, 1}	{1, 2, 1}	{1, 2, 3} {3, 2, 1}	{3, 2, 3}	{3, 3, 3}	{3, 1, 3}	{3, 1, 2} {2, 1, 3}	{2, 1, 2}	{2, 2, 2}	{2, 3, 2}	{1, 3, 2} {2, 3, 1}	{1, 3, 1}

In the aftermath of the proposed concepts, the different commutation states of a DMC can be set up in Table II with 6 rows and 12 columns, including twice the traditional stationary vectors ($2 \times 18 = 36$), and twice the rotating pairs ($2 \times 9 = 18$). Finally, the three traditional zero vectors ($\{1, 1, 1\}$, $\{2, 2, 2\}$, and $\{3, 3, 3\}$), strategically distributed through the remaining 18 positions to minimize commutations, complete the picture.

The layout of Table II has singular characteristics:

- 1) Not considering transitions between zero vectors and rotating pairs, every other transition among a cell (state) and its surrounding cells above, below, left or right, needs exactly one switch interchange.
- 2) This adjacency is circular, both in the horizontal or the vertical direction. For instance, row 1 is down below row 6 and column 12 is on the left of column 1.
- 3) Because of increased number of states, now the expanded SVM technique, based on the enlarged stationary vector set, seizes at least improved flexibility through one additional degree of freedom.

Besides the values of γ_n and ψ_k angles, the only difference between the rotating pairs equations in Table I and the stationary vectors equations (11) is the $\frac{2}{\sqrt{3}}$ factor in the latter. Yet, this is not a problem. Rather, this characteristic allows the slice of time of a rotating pair to be equally divided between both neighbor stationary vectors, as verified by (13), for n even:

$$\frac{2}{\sqrt{3}}e^{i\frac{\pi(n-1)}{6}} \times \frac{t_r}{2} + \frac{2}{\sqrt{3}}e^{i\frac{\pi(n+1)}{6}} \times \frac{t_r}{2} = e^{i\frac{\pi n}{6}} \times t_r \quad (13)$$

where t_r stands for any time allocated for a rotating pair.

III. ELABORATION OF A NEW EXPANDED SVM (ESVM)

The 72 switching states presented in Table II can synthesize any desired space vector transfer function using (9) format:

$$\begin{aligned} M &= m \times e^{i\gamma} \\ v_o &= \frac{\sqrt{3}}{2}m \times e^{i\psi} \times \text{Re}(e^{i\gamma} \times v_i) \\ W &= \frac{\sqrt{3}}{2} \times e^{i\psi} \end{aligned} \quad (14)$$

where m , γ and ψ modulation terms are free of choice. Applying the rectifying and inverting vector modulation method [3], angle γ offsets the "input" frequency from v_i , while ψ generates the v_o "output" frequency. Thus, angles γ and ψ

can be associated, for simplicity, to "input" and "output" sides respectively, even though MCs are in fact fully bidirectional.

Definition. Let $m_{k,n}$, for $k = 0 \dots 5$, and $n = 0 \dots 11$ be the (real) slice of time when (output) angle $\psi_k = k\frac{\pi}{3}$ is applied together with (input) angle $\gamma_n = n\frac{\pi}{6}$ during the commutation cycle. Let also be:

$$\alpha_{k,n} = \begin{cases} 1 & \text{when } \frac{n}{2} + k \text{ is even integer} \\ 0 & \text{when } \frac{n}{2} + k \text{ is odd integer} \\ \frac{2}{\sqrt{3}} & \text{otherwise} \end{cases} \quad (15)$$

The output voltage and input current are thus obtained by adding up the equations coming from Table II, per modulation cycle, in all possible angle combinations:

$$\begin{aligned} v_o &= \sum_{k=0}^5 \sum_{n=0}^{11} m_{k,n} \cdot \alpha_{k,n} e^{i\frac{\pi k}{3}} \times \text{Re}(e^{i\frac{\pi n}{6}} \times v_i) \\ i_i &= \sum_{n=0}^{11} \sum_{k=0}^5 m_{k,n} \cdot \alpha_{k,n} e^{-i\frac{\pi n}{6}} \times \text{Re}(e^{-i\frac{\pi k}{3}} \times i_o) \end{aligned} \quad (16)$$

Table II rows and columns will be numbered starting from 0 in order to match equations (16) notation.

As usual, both the ranges for input angle γ and for output angle ψ are divided in six $\frac{\pi}{3}$ intervals, and just vectors from the corresponding intervals are used each time. All other vector times are set to zero. Thus, Table II ends up divided in 36 six-vector sectors, which are used, one sector at a time, according to the required values of angles γ and ψ at each instant.

Examples of those sectors are highlighted in color aqua, beige and yellow in Table II. It must also be pointed out that:

- 1) Each sector comprises four stationary vectors, one zero vector and one rotating pair, which are enough to synthesize any transfer function in the sector's angle range.
- 2) Every sector overlaps its surrounding sectors in this way:
 - By three shared vectors above and below.
 - By two shared vectors on the left and on the right.
 - By one shared vector on each of the four corners.
which is useful to change sectors without discontinuities.
- 3) The rotating pairs are alternatively positioned on the upper side and on the bottom side of each sector.

In order to simplify calculations, angles γ and ψ are not used directly, but normalized to the first sector. Therefore, angles $0^\circ \leq \Psi \leq \frac{\pi}{3}$ and $-\frac{\pi}{6} \leq \Gamma \leq \frac{\pi}{6}$ are used instead.

The normalization law is:

$$\begin{aligned} \psi &= \Psi + K \times \frac{\pi}{3} & \text{for } 0 \leq K \leq 5 \text{ integer} \\ \gamma &= \Gamma + N \times \frac{\pi}{3} & \text{for } 0 \leq N \leq 5 \text{ integer} \end{aligned} \quad (17)$$

where K and N identify exactly one sector in Table II, which comprises rows K and $K + 1$ and columns $2N - 1$, $2N$ and $2N + 1$. Then, without any loss of generality, the slice of time for each of the six vectors in that sector can be settled using the mathematical formulas introduced in Table III, which can be verified by straight trigonometric calculations. Parameter $0 \leq m \leq 1$ is the traditional modulation index, as originally defined by Huber [1] and resulting in equation (19), whereas parameter $0 \leq \mu \leq 3$ tune the involvement of the rotating pair.

For instance, assuming an even N (rotating pair on top), the transfer functions related to vectors of (K, N) sector are:

$$\begin{aligned} &\frac{2}{\sqrt{3}} e^{i\frac{\pi K}{3}} \text{Re} \left(e^{i(\frac{\pi N}{3} - \frac{\pi}{6})} v_i \right) \quad e^{i\frac{\pi K}{3}} \text{Re} \left(e^{i\frac{\pi N}{3}} v_i \right) \quad \frac{2}{\sqrt{3}} e^{i\frac{\pi K}{3}} \text{Re} \left(e^{i(\frac{\pi N}{3} + \frac{\pi}{6})} v_i \right) \\ &\frac{2}{\sqrt{3}} e^{i(\frac{\pi K}{3} + \frac{\pi}{3})} \text{Re} \left(e^{i(\frac{\pi N}{3} - \frac{\pi}{6})} v_i \right) \quad 0 \quad \frac{2}{\sqrt{3}} e^{i(\frac{\pi K}{3} + \frac{\pi}{3})} \text{Re} \left(e^{i(\frac{\pi N}{3} + \frac{\pi}{6})} v_i \right) \end{aligned} \quad (18)$$

Following equation (16), the coefficients from Table III are applied to them and summed up, which delivers exactly:

$$v_o = \frac{\sqrt{3}}{2} m \times e^{i(\Psi + K \times \frac{\pi}{3})} \times \text{Re} \left(e^{i(\Gamma + N \times \frac{\pi}{3})} \times v_i \right) \quad (19)$$

Considering (17) γ and ψ angle normalization, then (14) space vector transfer function is properly synthesized. The very same equation (19) is obtained for the odd N sectors.

A. Rectifying and inverting vector modulation

Let $v_i(t) = v(t) \times e^{i \cdot w_i(t)}$ be a phasor representing the three-phase input system of a DMC, which could eventually be unbalanced and distorted. A theoretically "perfect" output sinusoidal system will be obtained, over a linear load, through DMC modulation. Thus, an output current phasor $i_o(t) = I_o \times e^{i(w_o \cdot t - \delta_{\text{cst}})}$ is desired, being I_o , w_o and δ_{cst} real constant values. Moreover, a displacement factor θ_{cst} is also required for the input current. A constant voltage $V_i \leq \min(v(t)) \forall t$

Table III
MODULATION COEFFICIENTS FOR EACH SIX-VECTOR SECTOR BASED ON
NORMALIZED ANGLES $0^\circ \leq \Psi \leq \frac{\pi}{3}$ AND $-\frac{\pi}{6} \leq \Gamma \leq \frac{\pi}{6}$

Sectors with rotating pairs on the upper side (aligned with 0°)			
$\Gamma \backslash \Psi$	0°	$\frac{\pi}{3}$	
$-\frac{\pi}{6}$	$m \left[\sin \left(\frac{\pi}{6} - \Gamma \right) - \frac{\mu}{6} \cos(3\Gamma) \right] \sin \left(\frac{\pi}{3} - \Psi \right)$	$m \sin \left(\frac{\pi}{6} - \Gamma \right) \sin(\Psi)$	
0°	$\frac{1}{3} m \mu \cos(3\Gamma) \sin \left(\frac{\pi}{3} - \Psi \right)$	$1 - m \cos(\Gamma) \sin \left(\Psi + \frac{\pi}{3} \right)$	
$\frac{\pi}{6}$	$m \left[\sin \left(\frac{\pi}{6} + \Gamma \right) - \frac{\mu}{6} \cos(3\Gamma) \right] \sin \left(\frac{\pi}{3} - \Psi \right)$	$m \sin \left(\frac{\pi}{6} + \Gamma \right) \sin(\Psi)$	

Sectors with rotating pairs on the bottom side (aligned with $\frac{\pi}{3}$)			
$\Gamma \backslash \Psi$	0°	$\frac{\pi}{3}$	
$-\frac{\pi}{6}$	$m \sin \left(\frac{\pi}{6} - \Gamma \right) \sin \left(\frac{\pi}{3} - \Psi \right)$	$m \left[\sin \left(\frac{\pi}{6} - \Gamma \right) - \frac{\mu}{6} \cos(3\Gamma) \right] \sin(\Psi)$	
0°	$1 - m \cos(\Gamma) \sin \left(\Psi + \frac{\pi}{3} \right)$	$\frac{1}{3} m \mu \cos(3\Gamma) \sin(\Psi)$	
$\frac{\pi}{6}$	$m \sin \left(\frac{\pi}{6} + \Gamma \right) \sin \left(\frac{\pi}{3} - \Psi \right)$	$m \left[\sin \left(\frac{\pi}{6} + \Gamma \right) - \frac{\mu}{6} \cos(3\Gamma) \right] \sin(\Psi)$	

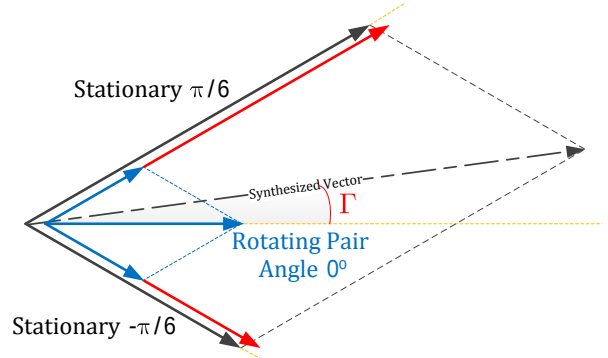


Fig. 2. Three vectors to synthesize input angle Γ .

should be chosen to avoid over-modulation. Since $m(t)$, $\gamma(t)$ and $\psi(t)$ (14) are free of choice, they are selected as:

$$m(t) = \frac{V_i}{v(t)} \quad \gamma(t) = \theta_{\text{cst}} - w_i(t) \quad \psi(t) = w_o \cdot t \quad (20)$$

Then, equations (14) and (9) are applied to get:

$$\begin{aligned} v_o(t) &= \frac{\sqrt{3}}{2} V_i \cdot \cos(\theta_{\text{cst}}) e^{i \cdot w_o \cdot t} \\ i_i(t) &= \frac{\sqrt{3} \cdot V_i \cdot I_o \cdot \cos(\delta_{\text{cst}}) e^{i(w_i(t) - \theta_{\text{cst}})}}{2 \cdot v(t)} \end{aligned} \quad (21)$$

Not only a fully distorted input system has turn into a sinusoidal output system, but an arbitrary displacement input factor θ_{cst} (capacitive or inductive) has been enforced as well.

B. Participation of the Rotating Pairs

The value adopted by μ in each commutation cycle calibrates the employment of the rotating pair, but have no influence on the synthesized equation (19). Thus, ESVM adds one degree of freedom to traditional SVM. Besides, any modulation law with independent input γ and output ψ angles, and involving rotating pairs, can be described by Table III formulations, so they represent the full ESVM capability. Some algebraic properties must be pointed out:

- $\mu = 0$ implies no use and $\mu = 3$ implies 100% usage of the rotating pair, that's why it must be $0 \leq \mu \leq 3$.
- The six elements of Table III sum exactly 1, and all of them are positive, regardless of Ψ and Γ , when $\mu \leq 2$.
- The range $2 < \mu \leq 3$ does not allow whatever Ψ and Γ combination, but it can be used to rule out one of the four stationary vectors, by zeroing its Table III coefficient.

The rotating pair only affects angle γ calculations, since it takes a bit from its two neighbor stationary vectors alone.

As can be seen in Fig. 2, the two black vectors are replaced by a blue rotating pair and two smaller red stationary vectors. Yet, this is not useful to improve the DMC voltage gain.

IV. CIRCULAR MODULATION USING ESVM

During a commutation cycle a specific six-vector sector is active, and within that sector, each vector should be applied, at least once according to its modulation coefficient, to synthesize the aimed transfer function. In addition, as explained in section II-D, the adjacency properties of Table II enable to cover that sector in a circular way with a minimal switching count.

The principle of circular modulation, illustrated in Fig. 3 for the example of the color aqua sector of Table II, implies:

- 1) Any vector can begin and/or end a commutation cycle.

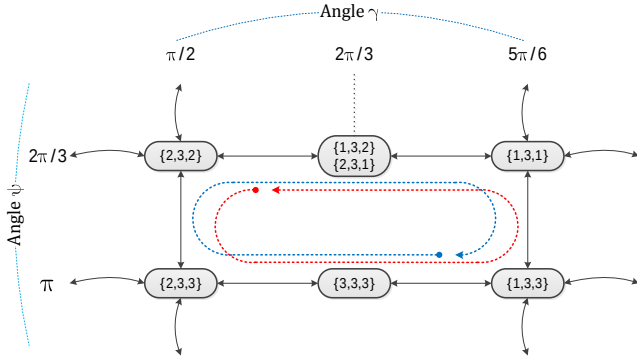


Fig. 3. Circular transitions among vectors in a sector.

- 2) Sectors can be covered either clock or counterclockwise, and even change direction during a commutation cycle.

The proposed circular modulation greatly improves flexibility while keeping the essence of the classic SVM technique.

It must be pointed out that the underlying philosophy of circular modulation aims to have each vector applied just once per modulation period, which means roughly five average transitions per cycle. A traditional sequence without rotating vectors [3] usually takes, back and forth, eight transitions (232 $\Rightarrow$ 233 $\Rightarrow$ 333 $\Rightarrow$ 133 $\Rightarrow$ 131 $\Rightarrow$ 133 $\Rightarrow$ 333 $\Rightarrow$ 233 $\Rightarrow$ 232 on Fig. 3 example) per cycle. Then, for the same switching frequency, the ESVM circular modulation waveform quality may seem worse because of fewer effective commutations, and not necessarily because of the rotating vectors usage.

A. Alternate Application of the Rotating Pairs

Table II was assembled such that each transition would take a minimal amount of commutations. However, even being able to work properly, a strict 2nd order SVM technique to synthesize the rotating pairs would involve a higher switching count, per commutation cycle, and will not be the preferred option hereinafter. Instead, each rotating vector of the pair will be alternately applied along the ensuing sampling periods.

In this way, regardless of the position of a vector in Table II, the switching count for any transition between vectors will always be exactly one, and 100% safe commutation will be achieved (see section V-B). As a drawback, a small distortion could be introduced for applying the two rotating vectors, with slightly different modulus, in successive sampling periods.

As a compromise, rotating pairs with determinants not exactly zero (namely, not purely stationary) are used in order to greatly ease vector transitions in every direction. Yet, section IV-B analytically discusses the possible distortion, and concludes that it quickly decreases with the sampling period, so being negligible for higher commutation frequencies.

B. Direct Alternate and Inverse Alternate Strategy

Since the technique described so far relies on (real or virtual) stationary vectors, the main concern of the alternate strategy for the rotating pairs is a potential deviation from its fixed position in space. Mainly, whether this strategy could introduce a cumulative error. Then, let's consider the couple of rotating vectors to build the rotating pair in position $(K, 2N)$:

$$e^{i\frac{\pi}{3}(K+N)}v_i \quad e^{i\frac{\pi}{3}(K-N)}(v_i)^* \quad (22)$$

Following (15), $(K+N)$ must be even. Properly normalized according to (17), any v_i can be set as $|v_i|e^{-i(\Gamma+N\frac{\pi}{3}-\theta)}$ in polar form, where θ is an arbitrary displacement input factor. Within a sector aligned with 0° (Table III) the contribution of each rotating vector for the pair in that position would be:

$$\begin{aligned} r_1(\Gamma, \Psi) &= \mu \frac{m|v_i|}{3} e^{i\frac{\pi K}{3}} e^{-i(\Gamma-\theta)} \cos(3\Gamma) \sin\left(\frac{\pi}{3} - \Psi\right) \\ r_2(\Gamma, \Psi) &= \mu \frac{m|v_i|}{3} e^{i\frac{\pi K}{3}} e^{i(\Gamma-\theta)} \cos(3\Gamma) \sin\left(\frac{\pi}{3} - \Psi\right) \end{aligned} \quad (23)$$

Just angles Γ and Ψ are affected by the alternate strategy because K is determined by the sector, $m|v_i|$ should be constant for a sinusoidal goal, and μ is also kept constant at least until the pair matching. It is clear that an ideal rotating pair $\frac{1}{2}(r_1+r_2)$ should follow the $e^{i\frac{\pi K}{3}}$ direction. The influence of small δ_Γ and δ_Ψ changes in angles Γ and Ψ respectively, along successive sampling periods, can be measured by:

$$\mathcal{E}_{da}(\delta_\Gamma, \delta_\Psi) = \text{Im} \left(\frac{3 \cdot e^{-i\frac{\pi K}{3}}}{\mu \cdot m \cdot |v_i|} \times \frac{r_1(\Gamma - \frac{\delta_\Gamma}{2}, \Psi - \frac{\delta_\Psi}{2}) + r_2(\Gamma + \frac{\delta_\Gamma}{2}, \Psi + \frac{\delta_\Psi}{2})}{2} \right) \quad (24)$$

which is proportional to the angle deviation from its right path.

Using trigonometric identities, $\mathcal{E}_{da}(\delta_\Gamma, \delta_\Psi)$ can be reduced to a linear expression involving $\sin(\delta_\Gamma \pm \frac{\delta_\Psi}{2})$ and $\sin(2\delta_\Gamma \pm \frac{\delta_\Psi}{2})$, which means that the angle deviation of the rotating pair will linearly decrease with the switching frequency when employing the direct alternate strategy.

Inverse Alternate Strategy: The error formula (24) is obtained applying always the clockwise and the counterclockwise vectors in the same order, namely $r_1, r_2, r_1, r_2, r_1, r_2, \dots$

The question arises whether inverting this order over ensuing rotating pair syntheses ($r_1, r_2, r_2, r_1, r_1, r_2, \dots$), could help to shrink this inaccuracy. Thus built, the pattern clock/counterclockwise repeats every four sampling periods, and the angle error $\mathcal{E}_{ia}(\delta_\Gamma, \delta_\Psi)$ due to the strategy would be:

$$\mathcal{E}_{ia}(\delta_\Gamma, \delta_\Psi) = \text{Im} \left(\frac{3 \cdot e^{-i\frac{\pi K}{3}}}{4 \cdot \mu \cdot m \cdot |v_i|} \times \left(r_2(\Gamma - \frac{3\delta_\Gamma}{2}, \Psi - \frac{3\delta_\Psi}{2}) + r_1(\Gamma - \frac{\delta_\Gamma}{2}, \Psi - \frac{\delta_\Psi}{2}) + r_1(\Gamma + \frac{\delta_\Gamma}{2}, \Psi + \frac{\delta_\Psi}{2}) + r_2(\Gamma + \frac{3\delta_\Gamma}{2}, \Psi + \frac{3\delta_\Psi}{2}) \right) \right) \quad (25)$$

Trigonometric calculations show that (25) turns exactly into:

$$\begin{aligned} \mathcal{E}_{ia}(\delta_\Gamma, \delta_\Psi) &= \frac{\sin(2\Gamma + \Psi + \frac{\pi}{6} + \theta)}{8 \cos(\delta_\Gamma + \frac{\delta_\Psi}{2})} \sin^2(2\delta_\Gamma + \delta_\Psi) \\ &\quad - \frac{\sin(4\Gamma + \Psi + \frac{\pi}{6} - \theta)}{8 \cos(2\delta_\Gamma + \frac{\delta_\Psi}{2})} \sin^2(4\delta_\Gamma + \delta_\Psi) \\ &\quad + \frac{\sin(2\Gamma - \Psi - \frac{\pi}{6} + \theta)}{8 \cos(\delta_\Gamma - \frac{\delta_\Psi}{2})} \sin^2(2\delta_\Gamma - \delta_\Psi) \\ &\quad - \frac{\sin(4\Gamma - \Psi - \frac{\pi}{6} - \theta)}{8 \cos(2\delta_\Gamma - \frac{\delta_\Psi}{2})} \sin^2(4\delta_\Gamma - \delta_\Psi) \end{aligned} \quad (26)$$

which is a remarkable outcome, with two key consequences:

- 1) Clearly, the higher the switching frequency, the lower δ_Γ and δ_Ψ angle changes, but unlike (24), now the angle

deviation of the rotating pair will quadratically decrease with the switching frequency as a result of (26).

- 2) The eventual cumulative error mentioned in section IV-B can be calculated as a mean integral of $\mathcal{E}_{ia}(\delta_\Gamma, \delta_\Psi)$. Whatever the relation between δ_Γ and δ_Ψ , which depends on the input/output frequency ratio, it will linearly lower with the sampling period as a result of (26) format.

This demonstrates that the *inverse alternate strategy* is better and should be preferred for ESVM and circular modulation.

C. Unmatched Pairs during Sectors Transition

The method of transition to neighbor sectors is seldom mentioned in the literature, taking for granted that some kind of discontinuity will often take place during that events.

However, as explained in section IV, the presented circular modulation has not a fixed beginning or ending vector. Then, the modulation can be dynamically adapted to always end at a border vector when the active sector is about to change.

Nevertheless, a question arises about matching of rotating pairs during a sector transition. Applying the alternate strategy (either direct or inverse), some pairs may eventually end up unmatched. Yet, this can be shown of no consequence:

- 1) When moving to a left or right sector, the modulation coefficient of the rotating pair, shown in Table III, will tend to zero because $\lim_{\Gamma \rightarrow \pm \frac{\pi}{6}} \cos(3\Gamma) = 0$. Therefore, eventual lack of matching will not matter.
- 2) Likewise, the modulation coefficient will also tend to zero when moving above or below through the zero vector side, because either $\lim_{\Psi \rightarrow 0} \sin(\Psi) = 0$ or $\lim_{\Psi \rightarrow \frac{\pi}{3}} \sin(\frac{\pi}{3} - \Psi) = 0$ depending on the sector type.
- 3) Finally, when moving above or below through the rotating pair side, the matching can be completed in the incoming sector, which the rotating pair also belongs to.

Thus, circular modulation with a rotating pair alternate strategy, ensures a constant one switching count at all times.

The Hidden Rotating Pairs: With a fully separate software layer taking care of the alternation, the rotating pairs applied through an alternate strategy, as developed in this paper, behave by all practical aspects, as if they were in fact real vectors. Including them, ESVM is a thirty vectors methodology.

D. ESVM Commutation Process

A circular graph, with 72 nodes, can represent the entire ESVM switching process. Fig. 3 illustrates part of that graph, where each node is a vector, as defined in section II-D, and each edge is a possible transition to an adjacent vector. Thus, the graph keeps being continuously scrolled, in an unending cycle, doing just one switch change at a time.

Within a specific sector, circular modulation involves just six (bidirectional) possible transitions among the six vectors of that sector. Moreover, Table II shows that those transitions always follow the same regular pattern in every one sector:

- 1) Circular modulation is discontinuous because each sector has an arm which does not commute at all.
- 2) Among the six effective transitions, three take place in each one of the other two arms which do commute.

- 3) All the input levels are always involved in each arm's transitions, namely $v_{i1} \leftrightarrow v_{i2}$, $v_{i2} \leftrightarrow v_{i3}$ and $v_{i1} \leftrightarrow v_{i3}$.

This uniform behavior may be harnessed in different ways.

V. PRACTICAL APPLICATIONS

Since rotating vectors produce zero common mode voltage, ESVM will always tend to improve that characteristic as a side effect. Moreover, as explained in section III-B, the parameter $2 < \mu \leq 3$ can also be used to maximize this outcome, even though this will not be the focus on the rest of the paper.

A. Improvement of Switching Losses

Power converter modulation methods have straight influence on commutation losses, and ESVM already ensures a minimal switching count, equal to one, for each vector transition.

Nonetheless, depending on the chosen path within the sector, losses may be further improved. The effective power losses in a specific commutation are roughly settled by the product $|\Delta v_i| \cdot i_o$, where Δv_i is the measured difference voltage level, and i_o is the measured current going through the arm.

The synthesis of a desired vector do not need to scroll all six transitions of the sector. Due to the flexibility of going back and forth, the worst transition can be avoided.

In fact, the total switching losses during a sampling period could be seen as a cost function, and the concepts of predictive control [18] might be applied to better define the effective path.

B. Robust Commutation

As deduced from section II formulation, each DMC transition must always involve two switches from the same column (arm): one to turn off and the other to turn on. Considering that commutations cannot be instantaneous in the real word, special bidirectional switches with two separate gates for each current direction are needed in order to avoid shorting the voltage sources or opening the current sources (see Fig. 1).

In any case, the voltage or current sign must be known to choose the right switching sequence. The four steps commutation is the most classical approach [30], with two style variations: voltage controlled and current controlled. However, both styles suffer from the same drawback since it is hard to determine the correct sign when Δv_i or i_o are close to zero.

As an improvement to the classical approach, a hybrid method was proposed in [31] through simple combination of both styles. Due to the uniform behavior of ESVM, verified in section IV-D, it is simply impossible for a given system, even just approximately sinusoidal, to have the event of simultaneous $\Delta v_i \approx 0$ and $i_o \approx 0$ occurring in more than one of the six transitions of a sector. Hence, adopting the hybrid method, circular modulation can easily avoid critical transitions, thus turning it 100% safe. Another robust (voltage style) switching method, based on modulation, has been presented in [32], but its application was restricted to unitary input power factor.

In fact, when there is not a critical transition in the sector, the switching losses improvement approach, presented in section V-A, might be followed as a subsidiary goal. Yet, this enhancement still needs further investigation.

VI. ESVM MODELING AND EXPERIMENTAL RESULTS

A high-fidelity model has been realized, using Modelica & SystemModeler[®], to verify the many different possibilities of the proposed technique. For common reference, the model, as well as the prototype experiment, are based on the next setup:

- Three-phase four-wire balanced input system, with $v_{ix} = 110 V_{RMS}$, $60 Hz$ in each phase.
- Star balanced load, $R = 30 \Omega$ and $L = 24 mH$ each arm.
- Second order input filter, with $C_f = 17.6 \mu F$ star connected capacitors, and $L_f = 2.0 mH$ series inductors.
- ESVM modulation parameters:
 - Converter switching frequency: $5 kHz$. In some plots, $7.2 kHz$ are employed to get better waveforms.
 - Modulation index (see Table III) $m = 0.9$.
 - 23.5° inductive displacement factor to offset the leading phase-shift imposed by the input filter.
 - Rotating pairs parameter $\mu = 0.8$ (see section III-B).
 - $16.5 V$ threshold for safe voltage commutation.
 - $0.5 A$ threshold for safe current commutation.

The functional block diagram is shown in Fig. 4. Under these conditions, Fig. 5 shows voltages and currents at the input side. Modulation (see section III-A) compensate the input filter phase-shift to drive the system input power factor close to one.

Likewise, Fig. 6 shows the variables at the load side when the output frequency is set to $90 Hz$. Just phase 1 switched voltage and 1 - 2 line voltages are shown in Fig. 6 in order to make the graphics readability easier. The 24.34° current phase-shift matches the specified load $\cos(\varphi)$ at that frequency.

In order to show in detail the ESVM vector sequence, Fig. 7 zooms in the involved voltages to the sampling period level. The slight differences between the grid and the output levels are due to the voltage drop in the input filter and in the matrix switches. Indeed, the two periods shown, where the rotating pair is highlighted, belong to the color aqua sector in Table II.

A. Safe Hybrid Commutation

As explained in section V-B, the algorithm will choose, in every step, either voltage or current commutation to avoid critical transitions, which means switching with Δv_i or i_o below the defined threshold levels. In order to measure the effectiveness of the technique, a safety ratio was introduced in the model, which evaluates to Δv_i divided by the threshold voltage if the commutation is voltage controlled, or to i_o divided by the threshold current, if it is current controlled.

Thus, this ratio sizes the safety margin in each commutation, with anything above 1 meaning that the commutation is safe. Fig. 8 shows the resulting safety ratio of the experiment.

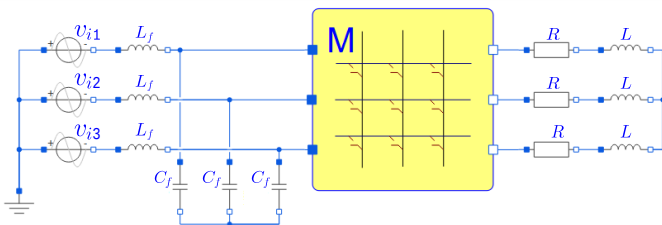


Fig. 4. Basic schematic of the model and the prototype setup.

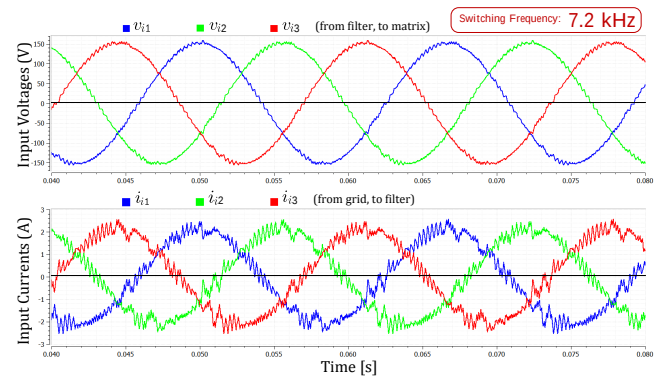


Fig. 5. Tensions v_{ix} fed to matrix, and currents i_{ix} supplied from the grid.

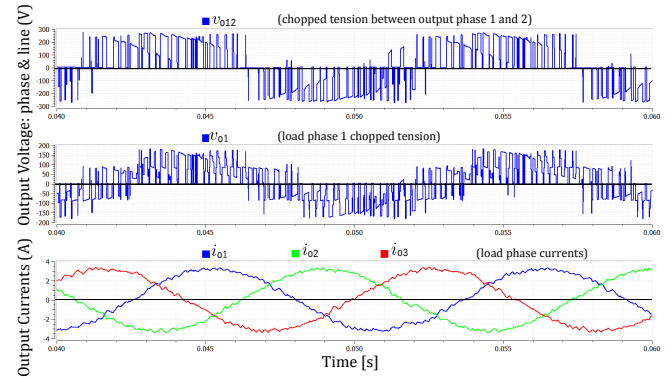


Fig. 6. Line v_{o12} , phase 1 v_{o1} voltages and i_{o1} , i_{o2} and i_{o3} load currents.

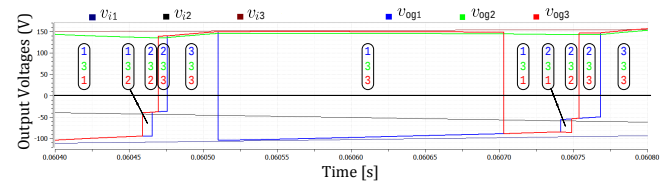


Fig. 7. v_{o1} , v_{o2} , v_{o3} referenced to system ground, and v_{ix} grid tensions.

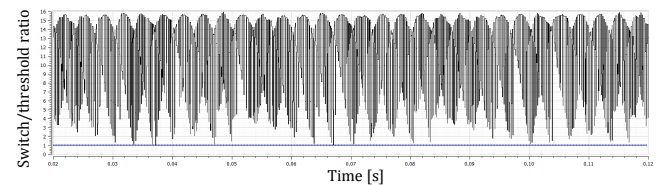


Fig. 8. Switching safety ratio using circular modulation.

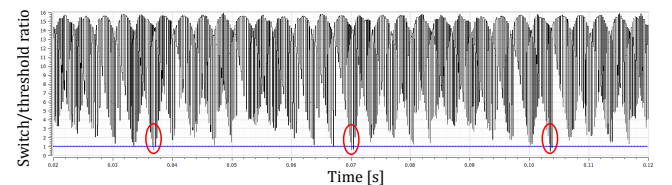


Fig. 9. Switching safety ratio with threshold level surveillance disabled.

When the threshold level surveillance mechanism of circular modulation is disabled, Fig. 9 shows three violations arising.

B. Prototype Experiment

In order to validate the proposed concepts and the simulation results experimentally, a laboratory prototype was built, as

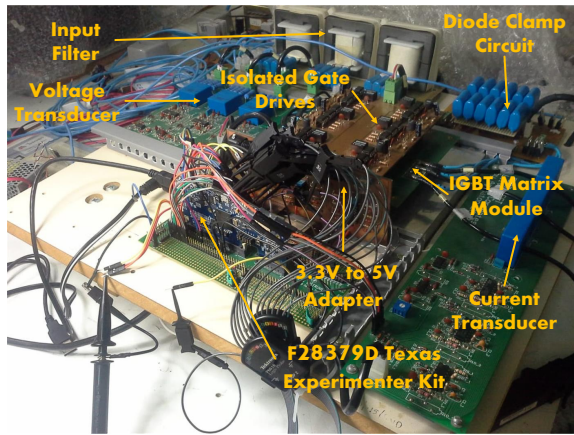


Fig. 10. Prototype assembly.

shown in Fig. 10. Besides the set-up of section VI, an Infineon IGBT FM35R12KE3 matrix module and a Texas F28379D Experimenter kit were key components. No FPGA was used.

The input voltages are measured by LEM LV-25P transducers, and the output currents are measured by LEM LA-55P transducers. Both are isolated hall-effect devices.

One EPWM module of the micro-controller takes care of the driving signals for each bidirectional switch, including the hybrid commutation schema referred to in section V-B, one co-processor (CLA1) is dedicated to the analog-to-digital acquisition system, and CPU1 is in charge of calculations and the ESVM circular modulation algorithm (section IV).

Nonetheless, half of the resources of the F28379D (CPU2, CLA2, memory) remained unused, available for a complementary higher level application, like a motor control for instance.

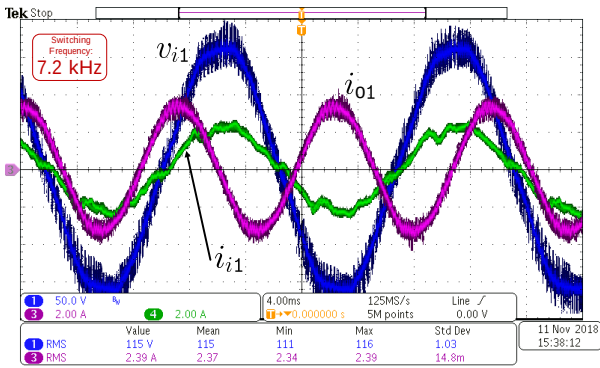


Fig. 11. Phase 1 60 Hz input tension and current, and 90 Hz output current.

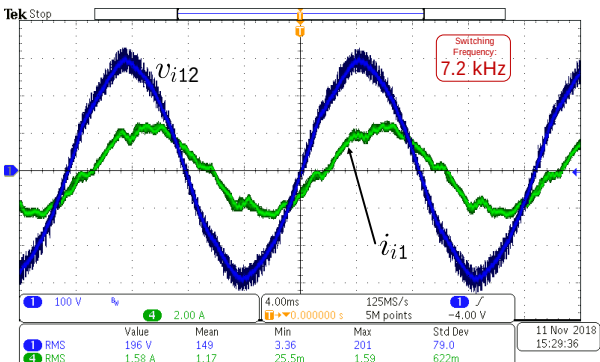


Fig. 12. Experimental results for input line voltage and input phase current.

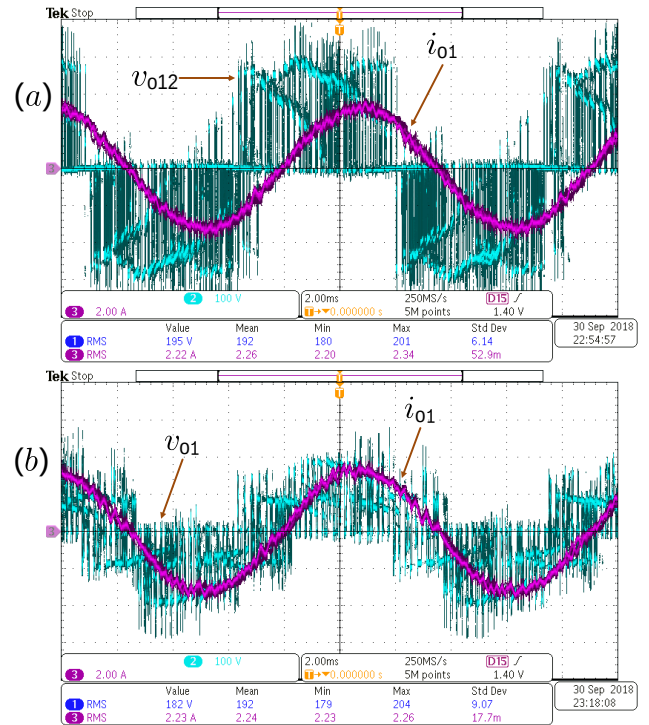


Fig. 13. Output line (a) and phase (b) voltages, output phase current (a & b).

Under the same operating conditions as earlier simulations, Fig. 11 shows input current and voltage, and output current. Despite the phase-shift introduced by the input filter, modulation makes the system appear as unitary power factor for the grid. Fig. 12 shows the input line voltage and the input phase current, now with a correct 30° phase-shift.

Fig. 13 shows the output variables, brought by the prototype. It can be observed that the experimental waveforms comply with those obtained by simulations in Figs. 5 and 6.

Waveforms of CMV using $\mu = 2$ and $\mu = 0$ (see section III-B) are shown in Fig. 15. Higher μ entails somewhat better CMV. Yet, middle μ results in slightly better current's waveforms because of fewer too-small generated vectors. Measured efficiency of the setup is roughly 97% (see Fig. 14).

C. Detailed ESVM Vector Sequence in the Prototype

As explained in section IV-D, and shown in Fig. 3, ESVM allows many ways to scroll through the vectors of a sector. During the commutation cycle, the algorithm will try to cover the sector (and synthesize the output vector) with a minimal number of vector transitions, which can range from 4 when no rotating pair is used (see section III-B), to 10 when aside from a critical transition to avoid, the position of the last vector to apply must be suitable for a sector change (see section IV-C).

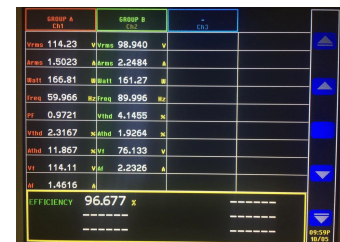


Fig. 14. Power analyzer data for section VI setup.

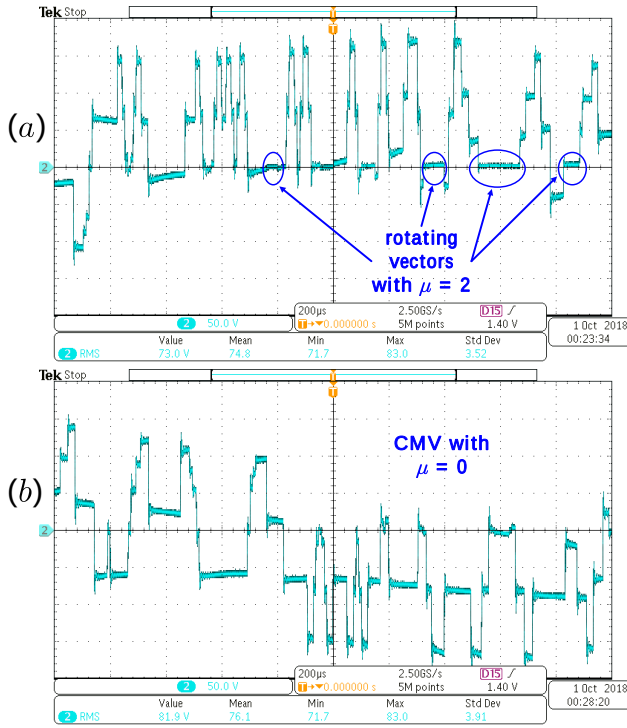


Fig. 15. Typical CMV with full (a) and no (b) rotating vectors participation.

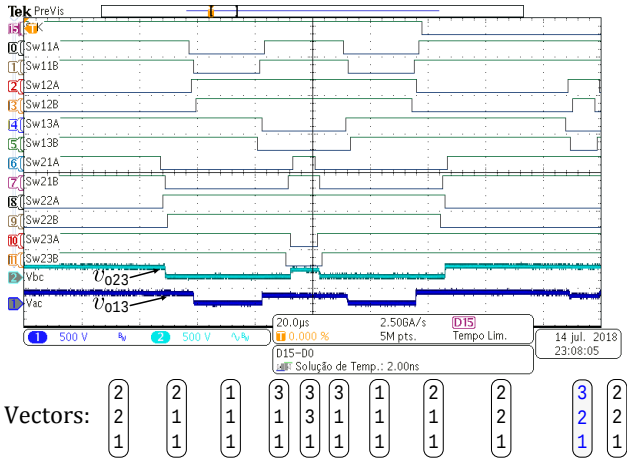


Fig. 16. Ten transition cycle: arms 1 & 2 active-low switch commands, output line voltages, and vector sequence with rotating vector highlighted in blue.

In order to catch a 10 transitions cycle, a spare pin was programmed to trigger the oscilloscope in one of such cycles. Fig. 16 shows a complete $200\mu s$ cycle, involving 10 vector transitions, which took place in the yellow sector of Table II.

Active-low gate signals correspond to switches in arms 1 & 2 as shown in Fig. 1, SwxxA runs the direct matrix to load path and SwxxB runs the reverse one. Arm 3 does not commute and is not shown, D15 is the oscilloscope trigger signal. The transition form $\{3, 3, 1\}$ to $\{3, 2, 1\}$ vector was avoided.

VII. CONCLUSIONS

This paper introduced the Expanded Space Vector Modulation (ESVM) for Direct Matrix Converters, based on the integration of the rotating vectors as rotating pairs. Besides from providing an additional degree of freedom, the new

vectors help to set up a highly connected circular graph which facilitate transitions both within a sector and between sectors.

It was shown that under the presented circular modulation method, the rotating pairs behave as if they were real vectors, provided that a fully independent software layer takes care of the alternation between the two rotating vectors of the pair.

On the basis of the new paradigm, different goals may be pursued, like better switching losses or robust commutation for instance. The quite uniform behavior of the method was used to ensure safe commutation by means of minimum threshold switching levels, either for Δv_i or i_o , which was verified through model simulations and prototype experimentation. The resulting waveform correctness has also been shown.

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